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Frege and the Functional Model of Sentential Complexity

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pelo Professor Doutor António Zilhão

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Resumo

Discuto três questões levantadas pela interpretação que Peter Geach faz do modelo funcional da complexidade frásica inventado por Gottlob Frege.

A leitura de Geach é orientada pela construção de soluções para três problemas sobre complexidade frásica. O primeiro é o problema de Ramsey, de saber como é que uma frase pode ter análises alternativas. O segundo é o problema da unidade frásica. O terceiro, o da unidade do pensamento.

O conceito fundamental da interpretação de Geach é o de uma função linguística. A solução do primeiro problema depende de conceber-se uma frase como o valor de uma função linguística; a do segundo passa por conceber um predicado como sendo ele próprio uma função; e a do terceiro, por conceber um pensamento como o valor de uma função-sentido.

Geach encontra latente nos escritos de Frege a noção de função linguística. A estratégia de Geach consiste em transferir para os níveis Fregeanos da linguagem e do sentido o modo de combinação que Frege advoga explicitamente para o nível da referência, no qual conceitos são para ser entendidos como funções de objectos para valores de verdade. O modelo proposto por Geach parece estar assim em consonância com o dictum de Frege segundo o qual podemos ver uma frase como um mapa de um pensamento. Deste modo, Geach oferece não apenas um tratamento uniforme dos três problemas mencionados, mas também uma representação sistemática das ideias de Frege.

Nesta investigação, pretendo testar o modelo de Geach, tanto no que toca a sua plausibilidade intrínseca, como relativamente à sua legitimidade exegética. Proponho uma modificação da solução de Geach para o problema de Ramsey; defendo a sua solução para o problema da unidade frásica; e afasto a unidade do pensamento como um mero pseudo-problema.

Palavras-chave

Funções linguísticas, decomposição e análise, relações conversas, paradoxo de Kerry, sentidos de expressões incompletas.

Abstract

I address three questions raised by Peter Geach's interpretation of Gottlob Frege's functional model of sentential complexity.

Geach's reading provides a solution to three problems about sentential complexity. The first is Ramsey's problem of how to make sense of a sentence's being analysed in different ways. The second is the problem of sentential unity. The third, that of the unity of thought.

The fundamental notion of Geach's account is that of a linguistic function. Geach's Fregean solution to the first problem depends on conceiving a sentence as the value of a linguistic function; to the second, on conceiving a predicate as being itself a function; and to the third, on conceiving a thought as the value of a sense-function.

Geach modestly traces the notion of a linguistic function to Frege's writings. Geach's strategy consists in extending to the Fregean realms of language and sense the mode of combination that Frege explicitly advocates to the realm of reference, in which concepts are to be seen as functions from objects to truth-values. Thus the model that Geach propounds accords well enough with Frege's dictum that we may regard a sentence as a mapping of a thought. And so Geach offers not only a unified solution to the three problems mentioned, but also a systematic representation of Frege's thought.

In this inquiry, I try Geach's model both for its intrinsic plausibility, and for its exegetical legitimacy. I propose a modification of Geach's solution to Ramsey's problem; defend his solution to the problem of sentential unity; and dismiss the unity of thought as a pseudo-problem.

Keywords

Linguistic functions, analysis and decomposition, converse relations, Kerry's paradox, the senses of incomplete expressions.

Resumo alargado

Discuto três questões levantadas pela interpretação que Peter Geach faz do modelo funcional da complexidade frásica inventado por Gottlob Frege.

A leitura de Geach é orientada pela construção de soluções para três problemas sobre complexidade frásica. O primeiro é o problema de Ramsey, de saber como é que uma frase pode ter análises alternativas. O segundo é o problema da unidade frásica. O terceiro, o da unidade do pensamento.

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Geach encontra latente nos escritos de Frege a noção de função linguística. A estratégia de Geach consiste em transferir para os níveis Fregeanos da linguagem e do sentido o modo de combinação que Frege advoga explicitamente para o nível da referência, no qual conceitos são para ser entendidos como funções de objectos para valores de verdade. O modelo proposto por Geach parece estar assim em consonância com o dictum de Frege segundo o qual podemos ver uma frase como um mapa de um pensamento. Deste modo, Geach oferece não apenas um tratamento uniforme dos três problemas mencionados, mas também uma representação sistemática das ideias de Frege.

Nesta investigação, pretendo testar o modelo de Geach, tanto no que toca à sua plausibilidade intrínseca, como relativamente à sua legitimidade exegética.

No primeiro capítulo, abordo a questão de saber se uma frase pode ser alternativamente representada em termos de argumento e função a partir do ponto de vista do chamado problema de Ramsey. Ramsey coloca o problema de saber como é que seria inteligível a existência de análises alternativas da mesma frase. Defendo que é a característica de que goza a noção de função linguística de generalizar a noção tradicional de predicado que permite tornar inteligível o fenómeno identificado por Ramsey. O facto de Frege aceitar que uma frase pode ter representações alternativas em termos de função e argumento constitui evidência a favor da atribuição da noção de função linguística a Frege por parte de Geach.

No entanto, proponho razões para crer que a concepção estrita de análise de Geach deve estar errada. A minha objecção ao ponto de vista de Geach pode ser formulada nos termos de Michael Dummett, que distingue análise de decomposição, da seguinte maneira: nem todos os componentes de uma frase estão imediatamente disponíveis por decomposição. Em geral, concordo com Dummett acerca da unicidade da análise. Mostro que as objecções típicas à interpretação que Dummett faz de Frege não são boas, e que os alegados contra-exemplos à unicidade da análise não podem senão ser ser entendidos, na melhor das hipóteses, como contra-exemplos a outra tese de Dummett, segundo a qual pensamentos Fregeanos são em geral para ser entendidos

como o sentido linguístico de frases, i.e. à ideia de que seria em princípo possível estabelecer uma bijecção entre pensamentos e frases da notação conceptual de Frege.

Por outro lado, algumas das críticas de Dummett ao ponto de vista de Geach são injustas. Concluo que a concepção de análise de Geach pode ser reformulada de maneira a incorporar o que de significativo existe na perspectiva de Dummett. Em particular, da unicidade da análise, ao contrário do que Dummett pensava, não se segue que predicados simples não sejam expressões incompletas.

No segundo capítulo, introduzo o problema da unidade frásica. Descrevo a solução que Geach apresenta para o problema, baseada na noção de função linguística. Mostro que a maneira como Frege descreve os seus "nomes funcionais" sugere fortemente a interpretação que deles faz Geach, segundo a qual eles devem ser insaturados no mesmo sentido em que funções são insaturadas. A insaturação dos predicados Fregeanos seria obviamente implicada pelo facto de eles serem funções linguísticas.

O paradoxo de Kerry, também conhecido como o paradoxo do conceito *cavalo*, é originado pelo facto de funções não poderem ser denotadas por nomes próprios. Explico por que razão é que o paradoxo de Kerry se segue da tese de que predicados são funções linguísticas. Por outro lado, esta mesma tese implica o reaparecimento do paradoxo de Kerry ao nível da linguagem. Com efeito, se funções não podem ser denotadas por nomes próprios, então, se predicados são funções linguísticas, então predicados também não podem ser denotados por nomes próprios, uma vez que funções linguísticas são funções. Geach argumenta que Frege foi capaz de prever este resultado. Alex Oliver, entre outros, discorda. Oliver tenta mostrar não só que a nota de "On Concept and Object" na qual Geach baseia o seu argumento é por ele mal interpretada, como também que Frege admite explicitamente que predicados podem ser denotados por nomes próprios. Admito que as críticas de Oliver à interpretação de Geach são boas críticas. No entanto, defendo que a conclusão mais razoável não é aquela que Oliver retira. Uma vez que a tese de que predicados são funções linguísticas implica o reaparecimento do paradoxo de Kerry ao nível da linguagem, mas que Frege não parece ter reconhecido tal fenómeno, Oliver conclui que, afinal, Frege não deve ter concebido predicados como funções linguísticas. A conclusão de Oliver é precipitada porque é também a tese de que predicados são funções que implica o próprio paradoxo de Kerry. Na sua ausência, portanto, ficamos sem uma explicação do motivo pelo qual Frege terá sido levado a considerar o paradoxo. Concluo assim simplesmente que a tese de que predicados são eles próprios funções deve ser atribuída a Frege, ainda que ele não tenha sido capaz de prever todas as suas consequências.

No terceiro capítulo, apresento o problema da unidade do pensamento por analogia com o problema da unidade entre conceito e objecto. Tal como a doutrina da insaturação resolve o regresso de Bradley no nível da referência, a extensão daquela doutrina para o nível do sentido soluciona o problema da unidade do pensamento. É essencialmente esta a solução de Geach para o problema da unidade do pensamento. No nível do sentido, são os sentidos de predicados que são funções. As "funções-sentido", como lhes chama Geach, são funções que, no caso mais simples, tomam sentidos de nomes como argumentos e geram sentidos de frases, i.e. pensamentos, como valores.

Dummett mostrou que a identificação dos sentidos de predicados com funções-sentido torna impossível a explicação do sentido de uma frase como a condição que deve ser satisfeita para que ela seja verdadeira. Este resultado é obviamente indesejável para um intérprete de Frege como Geach. Concordo com a crítica que Dummett faz a este aspecto do modelo de Geach. No entanto, defendo que a tese de que sentidos de predicados são funções-sentido não se segue da tese de que predicados são funções

linguísticas, e que portanto da negação da primeira não se segue a negação da segunda. Mostro que o problema da unidade do pensamento pode ser interpretado tanto como o problema da unidade frásica, como como o problema da unidade entre conceito e objecto. De outro modo, sugiro que o problema da unidade do pensamento é gerado ou por um equívoco, ou pela adesão a uma concepção demasiado esquemática da relação entre sentido e referência. As minhas conclusões são reforçadas pela hipótese de Jan Dejnožka segundo a qual, para Frege, sentidos não são nem funções nem objectos.

Finalmente, concluo que a generalização do modelo funcional da complexidade frásica proposta por Geach deve ser corrigida de duas maneiras. Em primeiro lugar, o modelo deve ser capaz de acomodar a distinção Dummettiana entre análise e decomposição, de acordo com a maneira entretanto descrita. Em segundo, o modelo não deve ser aplicado ao nível do sentido, uma vez que esta extensão é não apenas ilegítima, mas também desnecessária. O problema que a motiva é apenas um pseudo-problema.

Começo por descrever o método de análise frásica inventado por Frege no parágrafo 9 de *Begriffsschrift*, e por caracterizar a noção de função linguística de Geach, na Introdução.

Frege and the Functional Model of Sentential Complexity

José Manuel Mestre

Dezembro de 2014

À memória do Dr. Álvaro.

To perceive the words of understanding
Prov 1:2 KJV

Contents

Introduction

When Frege asked himself what could he regard as the result of his work, this is what he had to say.

> It is almost all tied up with the concept-script, a concept construed as a function, a relation as a function of two arguments, the extension of a concept or class is not the primary thing for me. unsaturatedness both in the case of concepts and functions, the true nature of concept and function recognized.[1]

Frege's doctrine of unsaturation can be seen as an answer to Bradley's regress.[2] The extension of this doctrine from the realm of reference to those of language and sense has allowed Peter Geach to answer three puzzles about sentential complexity.

The first is "Ramsey's problem", of whether a single sentence may intelligibly be thought to have alternative analyses. The second is the old Platonic problem of sentential unity. The third is the problem of the unity of thought.

Geach's solutions rely on his notion of a linguistic function, the discovery of which he attributes, even if only implicitly, to Frege. Thus Geach provides not only a unified account of all three puzzles, but also a systematic interpretation of Frege's thought, since he modestly traces his views essentially to Frege's texts. Indeed, on the model propounded by Geach, Frege's idea that sentences might serve as pictures both of their senses and, to a certain extent, of their referents, is admirably interpreted.

Our task is twofold. We will try Geach's account both for the intrinsic plausibility of his solutions, and for its exegetical legitimacy, eventually to conclude for the need of a few corrections.

In the first chapter, we will see how the notion of a linguistic function allows us to interpret Frege's function names as generalizing the traditional notion of a predicate. Even if analysis, as Dummett has claimed, is unique, it is still necessary to account for the intelligibility of there being alternative decompositions of sentences. It is the generality of the notion of a linguistic function that provides for the latter.

In the second chapter, we will see that it is the fact that linguistic functions are themselves unsaturated that suggests a solution to the problem of sentential unity. In connection with Kerry's paradox of the concept *horse*, we will see that, although Frege required his function signs to be incomplete, he did not draw the full consequences therefrom.

In the third chapter, we will see that Geach's solution to the problem of the unity of thought involves an implausible account of the senses of incomplete expressions. I will argue that the problem of the unity of thought is after all only a pseudo-problem. However, *contra* Dummett, Geach's solution to the latter is independent from his general account of analysis.

At the heart of all these matters lies of course Frege's celebrated method of analysing sentences in terms of function and argument. This, he introduced in his *Begriffsschrift*.

[1] Frege 1906b: 184.
[2] See chapter 3.

0.1 *Begriffsschrift* §9

> Suppose that a simple or complex symbol occurs in one or more places in an expression (whose content need not be a possible content of judgment). If we imagine this symbol as replaceable by another (the same one each time) at one or more of its occurrences, then the part of the expression that shows itself invariant under such replacement is called the function; and the replaceable part, the argument of the function.[3]

Frege's procedure of function-argument analysis may be characterized in his later terminology as an operation to form function-names from either proper names (which include definite descriptions and sentences) or function-names. It basically corresponds to the *inverse* process of his "two ways to form a name" from *Grundgesetze*. Here Frege writes that there arises

> [A] a proper name
> [1] from a proper name and a name of a first-level function of one argument,
> or [2] from a name of a first-level function and a name of a second-level function of one argument,
> or [3] from a name of a second-level function of one argument of type 2 and the name […] of a third-level function;
> [B] the name of a first-level function of one argument
> [1] from a proper name and a name of a first-level function of two arguments.[4]

Michael Dummett called the operation described by Frege 'extraction of functions'. But since it concerns the formation of function-names, i.e. what we now call predicates, I should rather prefer to say 'function-name extraction' or 'predicate extraction', after Peter Sullivan's 'predicate abstraction'. Sullivan's term is unfortunate only because it lends itself to confusion with abstractionism – the very theory of concept-formation that Frege's own came to replace.

It may or may not be the case that function-names are themselves functions: *linguistic* functions, as Geach would claim them to be.[5] Frege's account of predicate extraction does imply that predicates are functions. In fact, he *defines* functions as predicates: a function just is that part of an expression that shows itself invariant under the replacement of some other part. But then carelessness about sign and thing signified abounds in *Bs*. For instance, when Frege introduces his notation for the expression of generality, he writes that the judgement-stroke, followed by a content-stroke in which a concavity has been inserted with a Gothic letter on top of it, followed by a function of the same Gothic letter, is to signify "the judgement that the function is a fact whatever we take its argument to be."[6] Now if a function is an expression it cannot be a fact in the intended sense. Frege was here guilty of the "mistake, […] that is very often met with in mathematical works, even those of celebrated authors",[7] which he eventually diagnosed as one of the causes of the malady of formalist philosophies of arithmetic. So in all probability he made the same mistake in *Bs* §9.

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[3] Frege 1879 §9.
[4] Frege 1893 §30. Note that the type of an argument is not its level but its arity: an argument of type 1 is an object; an argument of type 2 is a first-level function of one argument; of type 3 is a *first*-level function of two arguments. See Frege 1893 §23.
[5] See chapter 2.
[6] Frege 1879 §11.
[7] Frege 1891: 2.

In any case, regardless of whether function-argument analysis is misleadingly so-called, predicate extraction *is* an operation to form function-*names*. That much is clear from Frege's later account in *Gg*, which is posterior to his distinction between sense and reference, after which Frege became hypersensitive to the distinction between use and mention. In *Bs* Frege may have used 'function' where he should have 'function-name', but then he *should* have 'function-name': not 'function', not 'sense'. That is to say that function-argument analysis as introduced by Frege essentially operates on the realm of language.

As Geach would say, to analyse a sentence in terms of function and argument is equivalent to discerning a pattern within it. It is easy to display the patterns instantiated by a sentence by a simple change in font style. Consider 'Cato killed Cato', to use Frege's own example.[8]

Cato **killed Cato**.
Cato killed Cato.
Cato **killed** Cato.

In each token of the sentence we can now imagine as replaceable, say, the part written in regular font style, and as invariant the part in bold. Once the patterns are so recognized, it is a short step to see that each may in turn be instantiated by other sentences.

Cato **killed Cato**.	**Cato killed** Cato.	Cato **killed** Cato.
Brutus **killed Cato**.	**Cato killed** Brutus.	Brutus **killed** Brutus.
Caesar **killed Cato**.	**Cato killed** Caesar.	Caesar **killed** Caesar.

And so from 'Cato killed Cato' we can extract the names of the first-level functions:

( ) killed Cato
Cato killed ( )
( ) killed ( )

and, by imagining now the bold part as variable, the names of the second-level functions:

Cato ( )
( ) Cato
Cato ( ) Cato.

These functions may be read respectively as: the property of having killed Cato; the property of being such that Cato killed it; the property of killing oneself; the property of being a property of Cato; and the property of being a relation holding between Cato and himself.[9]

The brackets mark the argument-places of the function-sign, thereby indicating where the sign for the argument must in each case be inserted in order for a proper name, i.e. a sentence, to be formed. Dots or dashes could have been used for the same purpose. Geach aptly describes the role of function-names as that of serving as

[8] Compare this explanation with Sullivan's (2004: 694-6).
[9] Frege 1879 §10.

"stencils" for constructing other expressions.[10] However, this notation becomes impractical when Frege's procedure is applied to predicates themselves.

> Suppose that a symbol occurring in a function has so far been imagined as not replaceable; if we now imagine it as replaceable at some or all of the positions where it occurs, this way of looking at it gives us a function with a further argument besides the previous one. In this way we get functions of two or more arguments.[11]

We can indeed imagine 'Cato' as replaceable in '( ) killed Cato'

( ) killed **Cato**

thus recognizing that the sentence 'Cato killed Cato' shares a pattern also with 'Brutus killed Caesar'. The new function-name could be represented as '… killed ---'.

The need to distinguish perspicuously between different argument-places led Frege to introduce a special notation for the purpose. He chose small-case Greek consonants, which do not belong to the vocabulary of his concept-script, to mark argument-places, so that the function-name 'ξ killed ζ' could now be clearly distinguished from 'ξ killed ξ'. To the places occupied by, say, 'ξ', Frege called 'ξ-argument-places'. In 'ξ killed ξ', the ξ-argument-places are said to be *related* to one another; in 'ξ killed ζ', the ξ-argument-place is *not* related to the ζ-argument-place.[12] Related argument-places *must* be filled by the same arguments; unrelated argument-places *may* be filled by the same or by different arguments.

A function may have only one argument, and yet its name several argument-places. If a function has two arguments, then its sign must have unrelated argument-places, to indicate that once one of the argument-places is filled, we are still left with a function-name (of one argument-place) standing for a function of one argument. Functions with one argument whose values are always truth-values are called concepts, and their names concept-words;[13] functions with two arguments whose values are always truth-values are relations, and their names relation-words.[14] Thus 'ξ killed Cato' is a concept-word, 'ξ killed ξ' a concept-word with two (necessarily related) argument-places, and 'ξ killed ζ' a relation-word.

José Luis Bermúdez has accused Frege of "overlooking" the fact that the two-place predicate 'ξ killed ζ' could also be extracted from 'Cato killed Cato', and so of not distinguishing the former from 'ξ killed ξ'.[15] But as a matter of fact, it has been Bermúdez who neglected that Frege's procedure only authorizes the extraction, from sentences, of names of functions of one argument (i.e. monadic predicates or predicates with related argument-places). Function-names with unrelated argument-places must be extracted from other function-names. Thus 'ξ killed ζ' must necessarily be obtained only either from 'Cato killed ξ' or from 'ξ killed Cato', after a second application of the operation. Indeed, as Frege had said at the beginning, the subject of predicate extraction "need not be a possible content of judgement".

Now, instead of 'Cato', we could have chosen to imagine as variable 'killed' in '( ) killed Cato' in order to arrive at '… --- Cato'. But using the same notation as

[10] Geach 1976b: 59.
[11] Frege 1879 §9.
[12] Frege 1893 §4.
[13] Frege 1893 §3.
[14] Frege 1893 §4.
[15] Bermúdez 2001: 10, note 12.

before and writing the function-name as '$\xi\ \zeta$ Cato' would have the undesirable result of allowing us to form such strings as 'Cato Cato Cato' or 'Brutus Caesar Cato', which are not sentences. Since '… --- Cato' is the name of an unequal-level function, its argument-places must be filled by argument-signs for signs of different levels. Frege introduced a further refinement in his notation in order to indicate not only that different argument-places may be unrelated, but also that different argument-places may be places for signs of arguments of different levels.

As a limiting case of predicate extraction, we have the case in which the whole sentence is imagined as variable. Thus 'ξ' would be the name of the function whose value for each sentence as argument is the sentence itself. This is indeed useful to identify names of truth-functions. 'Cato killed Cato and ξ' can be extracted from 'Cato killed Cato and Brutus killed Caesar', and then 'ξ and ζ' from the former. Now it is a shame that Frege allowed 'ξ' to be filled by proper names.

Indeed Frege did not distinguish this case from that in which 'Cato' is imagined as replaceable in the expression (whose content is *not* a possible content of judgement) 'Cato'. Hence 'ξ' would also be the name of the function whose value for each object as argument is that very object. Frege distinguished neither ξ from identity, nor predicates from operators. As a result, Frege might have recognized 'Cato and Brutus' as a possible value for conjunction (as a *propositional* connective) and 'Cato killed, Brutus killed Brutus' as a possible value for 'Cato killed ξ'. Such sentences were not nonsense but false.

It is amusing that it was the fact that Frege did not distinguish the functions mentioned that led him not to distinguish their names, and not otherwise. Sentences were for Frege only a kind of complex proper names of a peculiar sort of logical objects, and so belonged to the same logical category as proper names. Thus 'Cato killed Cato' and 'Cato' could be arguments (and values) of 'ξ'. Ironically, Frege believed this to be a virtue of his doctrine for technical reasons. This may have been a result of his confused theory of sense and reference which involves two distinct notions of reference: one which is based on the name/bearer relation as prototype, and one which coincides with the notion of semantic value. It is not yet problematic to say that names denote objects, and sentences truth-values, as long as the notion of reference involved is that of semantic value. Now, in case of proper names, the two notions coincide: the bearer of a proper name is its semantic value. But in the case of sentences, while their semantic values are indisputably truth-values, it is an awkward thing to say that the latter are the bearers of sentences, or indeed that sentences have bearers of any kind.

So in *Gg* Frege retracted from the healthy doctrine of *Bs* that a possible content of judgement must always be complex. In *Gg*, though not in *Bs*, proper names are allowed to follow the content-stroke. The sign '—Cato' represents the content that Cato is a name of a truth-value; the sign '⊢—Cato' represents the false assertion that Cato is a name of the True. Frege's mature doctrine is committed to a highly implausible account of assertion as an intentional act.[16] But it could have been easily avoided by recognizing the distinctive logical roles of proper names and sentences, and by devising appropriate notations for objectual identity and propositional identity. Frege lacked the insight that had led him to distinguish between first- and second-level functions and the names thereof.

Predicate extraction is constrained in three ways. First, not every pattern that a sentence may instantiate is a relevant pattern for predicate extraction. 'Catξ' is not a

[16] Geach 1976b: 62.

predicate shared by 'Cato killed Cato' and 'Cat is mammal'. This shows that the relevant kind of complexity is not of the typographical kind. It is presupposed by predicate extraction that one knows the language: which are the smallest meaningful bits of language that circumscribe the different parsings of the same sentence. Second, not every grammatically relevant pattern is logically relevant. 'ξ is wise' and 'ξ has the property of being wise' are to count as the same predicate.[17] Finally, predicate extraction also presupposes knowledge of the category to which an expression may belong; and so an insight into level-distinctions is required in its application.

0.2 Linguistic functions

Peter Geach has interpreted Frege's function-names as being themselves functions of a linguistic sort.

> The function $2.\xi^2+\xi$ is a numerical function; i.e. it takes numbers as arguments, and its value for a number as argument is again a number. But the values and arguments of a function need not be restricted to being numbers. A kind of non-numerical function that it will be specially useful to consider is a *linguistic* function (as we may call it); a function that takes names as arguments, and whose value for a name as argument is again a (complex) name.[18]

A linguistic function is therefore a function from names to names (in Frege's extended sense of a name), just as, say, a concept is a function from objects to truth-values, and numerical functions in general functions from numbers to numbers. A function from numerals to numerals would of course be a linguistic function. Whether there are linguistic functions is not the point, since they can simply be so defined: the question is rather whether function-names are such functions, and whether Frege thought so too.

According to Geach, "it seems right to say" that what represents in language the function $2.\xi^2+\xi$ is the linguistic function whose values for the numerals '2', '3' and '7' are respectively the complex names '2.2^2+2', '2.3^2+3' and '2.7^2+7'. This amounts to identifying the name of the function $2.\xi^2+\xi$ with the common pattern instantiated by the complex names

2.$2^{\mathbf{2}}$**+**2
2.$3^{\mathbf{2}}$**+**3
2.$7^{\mathbf{2}}$**+**7.

Now a pattern instantiated by a sentence is a pattern according to which the sentence may be conceived as having been constructed. A linguistic function is therefore a rule for the construction of expressions. A function-name indicates such a rule by serving as a "stencil" for forming expressions. Geach cites as an example of the predicate 'ξ killed ζ' the rule:

> The name x is written just before, and the name z just after some token of the word 'killed'.[19]

[17] We are to see that the difference between them would be a difference of tone, not of sense.
[18] Geach 1961: 143.
[19] Geach 1975: 148.

At this junction, one is met by "a certain linguistic awkwardness, of some philosophical interest."[20] We want to call the expression '$2.\xi^2+\xi$' the linguistic function that represents $2.\xi^2+\xi$. But the quoted words do not actually occur in the expressions in which the linguistic function is said to occur, e.g. the complex names '2.2^2+2', '2.3^2+3' and '2.7^2+7'. Likewise, we speak of the predicate 'ξ killed ζ' as occurring in 'John killed Mary' and in 'Mary killed John', but again what we quote does not figure in these sentences. The reason is simple enough. As Frege himself notes,

> […] when we say 'the function $1 + \xi - \xi$', the letter 'ξ' is not part of the function-sign; for the proper name '$1 + 3 - 3$' is composed of the function-name and the proper name '3', and the letter 'ξ' does not occur in it at all.[21]

Geach concludes that, "As Frege would put it, by a kind of linguistic necessity we cannot quite say what we are trying to say."

> The actual expression 'ξ killed ζ' is neither a function nor a predicate: it serves however to identify a two-place predicate shared by many sentences, and this is the same thing as identifying a function yielding such sentences as 'John killed Mary' and 'Mary killed John' as its values when proper names are supplied as its arguments.[22]

Alex Oliver has criticized Geach for here revealing too narrow an account of quotation.[23] According to Geach, the phrase 'the predicate "ξ killed ζ"' denotes, not a linguistic function, which would be the intended predicate, but an actual expression, which is not. Instead, the intended linguistic function is indicated, or identified, as Geach says, by such an expression as 'ξ killed ζ'. But then we can denote the linguistic function directly by the roundabout phrase 'the linguistic function, or two-place predicate, indicated by the actual expression "ξ killed ζ"'. The circumlocution is, however, completely superfluous. Oliver compares Geach's position to that of logicians who often devise different styles of quotation marks to avoid ambiguity as to what they are quoting: type or token expressions, schemata, non-linguistic items such as concepts, etc. Oliver cites the Kneales with approval, for whom "Quotation marks were made for man, not man for quotation marks."[24] We can avoid the ambiguities in more relaxed terms, for instance by the use of an auxiliary prefix, as in 'the printed token "loves"', 'the phonological type "loves"', 'the lexeme "loves"', etc.[25] So if the phrase 'the predicate "ξ killed ζ"' does not denote a linguistic function, we might as well use 'the linguistic function "ξ killed ζ"', which does.

Oliver's critique would already sound unfair to someone as Geach, who, concerning the King James Bible, wrote:

> 'God said "Let there be light"' does not mean that God used English, and only a philosopher would argue that this is a mistranslation unless the original consisted of a quoted English sentence standing in a Hebrew context.[26]

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[20] Geach 1975: 148.
[21] Frege 1914a: 239.
[22] Geach 1975: 149.
[23] Oliver 2010: 124.
[24] Kneale and Kneale 1962: 514.
[25] Oliver 2010: 124.
[26] Geach 1967: 167.

But it would nevertheless be a good objection if the phrase 'the linguistic function "ξ killed ζ"' did refer to the intended linguistic function – but it doesn't, and so it isn't. *That*, precisely, is the linguistic awkwardness.

What we have here, as Geach says, is a reduplication of Frege's paradox of the concept *horse* on the linguistic level.[27] The problem is not one about quotation at all. What is in question is how to refer to linguistic functions, which of course is only an instance of how is it possible to refer to functions in general.

For someone who did not believe that predicates were linguistic functions, or that functions were unsaturated, or that one could only refer to functions by function names, as Oliver legitimately might, there would surely be no question as to whether either 'the predicate "ξ killed ζ"' or 'the linguistic function "ξ killed ζ"' could be used to refer to their intended meanings: they would. But the "linguistic awkwardness" recognized by Geach is not a premise in his argument for the conclusion that predicates are linguistic functions: it is a consequence of the latter.

Oliver has also claimed that both Frege and Geach were rather hasty to conclude that Frege's Greek consonants were not part of his function signs only because they did not figure in the expressions in which the function signs were said to occur. True, 'ξ' figures in 'ξ killed ζ' but not in 'Mary killed John'. But it all comes down to what it means for an expression to occur in another.[28]

As Oliver points out, what it means for the schema '*Fx*' to occur in the sentence '*Fa*' is that '*Fa*' is just the result of substituting a term, in this case '*a*', for '*x*' in '*Fx*'.[29] Therefore, at least in the case of schemata, a proper part of an expression may not be a proper part of the sentence in which the expression occurs.

Geach's argument appears to be (i) the predicate 'ξ killed ζ' occurs in 'Mary killed John'; but (ii) the actual expression, or "the actual bit of print" 'ξ killed ζ', does not; because (iii) 'ξ' is a part of the expression but not of the sentence; therefore, (iv) the predicate is not the actual expression, but is only indicated by it.

Now one can conceive of 'ξ killed ζ' as a schema. But then premise (ii) is false. The schema, which according to Oliver is an actual expression, does occur in 'Mary killed John'. 'To occur' does not necessarily mean 'to be an integral part of': Geach and Frege simply miss the relevant sense.

It is not immediately clear what to make of this objection of Oliver's. I suspect that Geach would simply reply that, in the exact measure in which a schema is said to occur in a sentence, it is not an actual expression, *pace* Oliver. In other words, a schema, in the sense described, is a linguistic function. So it is false that (ii) is false.

Oliver himself implies that the notion of a linguistic function only generalizes the notion of a schema when he distinguishes both.

> Instead of using an '*x*' in a schema, others might use a '*y*' or dots or lines or circled numerals or Greek consonants […]. The choice is plainly arbitrary, but it prompts a search for a sense of pattern according to which these trivially different schemata all depict the same pattern. Our sense of pattern gives us what we want, since the same linguistic function may be described in terms of substitution of a term for e.g. the letter '*x*' in '*Fx*' or the dots '…' in '*F*…'. Of course, it may also be described without making

[27] Whether Frege "foresaw" it, and whether he thought predicates to be incomplete expressions in Geach's sense, will be discussed in chapter 2 below.
[28] Oliver 2010: 124.
[29] Oliver 2010: 120.

> any reference to a schema, e.g. *the result of attaching the term … to the expression 'F'*.[30]

This relates to a point Oliver later makes when he considers objections against the view that predicates may be plain expressions. Oliver says that

> Nothing changes if expressions are individuated more finely, by building grammatical rules into their identity conditions. It would still be true that both kinds of predicate may be construed as expressions.[31]

He had previously characterized plain expressions as type expressions with a certain grammar, that "dictates that [the predicate] occurs only once in an *atomic* sentence". Oliver writes that

> The grammatical rule governing the predicate '*F*' invokes a general method of constructing sentences, namely *predication* of a one-place predicate […]. This method of construction may be identified with the *linguistic* function that maps two arguments – a one-place predicate Φ and a term α – to a single value – the sentence $\Phi\alpha$.[32]

What is more, a plain expression cannot be an expression *simpliciter*, but must essentially involve its grammar:

> a predicate needs to be distinguished from its homophones and homographs, which may not obey the same rules of combination. In other words […] a simple predicate cannot be an expression *simpliciter*, but only an expression coupled with a grammatical rule.[33]

The implication would be that predicates do not become linguistic functions, even when grammatical rules are "built in their identity conditions". But when Oliver compared the linguistic function that dictates the behaviour of a plain expression with a linguistic function in general, he had said the following:

> The linguistic function *f* is related to, but distinct from, the linguistic function that I earlier identified with the predication construction. The latter takes *two* arguments, e.g. it maps '*F*' and '*a*' to '*Fa*', whereas *f* takes just *one*, e.g. it maps '*a*' to '*Fa*'. In effect, the expression '*F*' has been absorbed into the function *f*.[34]

So a linguistic function seems to be as much a generalization of "expressions *simpliciter* plus rules" as it is of a schema. This makes both Oliver's plain expressions and schemata particular cases of linguistic functions alike. The reason that the same linguistic function may be alternatively specified is precisely that the notion of a linguistic function is more general than that of a grammatical rule.

The function-name of the function whose "value for every argument coincides with the argument itself" – that is, identity – is simply 'ξ'.[35] According to Oliver's definitions, it could not count as a plain expression, since it has "no separable bit of print that anybody might take as playing [the] role of a functor."[36] But it could count as

[30] Oliver 2010: 120-1.
[31] Oliver 2010: 126.
[32] Oliver 2010: 119.
[33] Oliver 2010: 126.
[34] Oliver 2010: 121.
[35] Frege 1893 §26.
[36] Geach 1976b: 59.

a limiting case of both schemata and linguistic functions. So the latter are more general than plain expressions. Oliver comments thus on a similar case (discussed by Harold Noonan, that of the function-name 'ξ^{ζ}'):

> It is true that '2^{4}' contains no expression standing for the function involved. It features only an *arrangement* of the terms '2' and '4'. But from the facts about this particular case it hardly follows that *no* functor, actual or possible, can be construed as an expression. When an 'auxiliary' expression does happen to be present, as in '2 raised to the power 4', it is a serviceable candidate for the relevant functor. Again, that there is a *possible* language in which no functor is an expression […] is compatible with the fact that in *our* language expressions comprise one kind of candidate for many functors. As for functors, so for predicates.[37]

I cannot see the point of Oliver's comment. It *does* follow from the fact that 'ξ^{ζ}' *may* be a function-name that, in general, predicates *need not* be construed as expression. If it is not the case that a predicate is construed as an expression in every possible language in which it occurs, then the predicate is not essentially an expression: it is one only contingently. Given that there are (arguably) no contingent identities, it follows that predicates are not expressions, since they are not essentially expressions.[38]

I conjecture that Oliver doesn't see that the notion of a linguistic function just is a generalization of plain expressions and schemata to the point of covering even such cases as 'ξ^{ζ}' and 'ξ'. Interestingly enough, it is the fact that linguistic functions generalize in another sense the traditional notion of a predicate that will be our initial focus in the first chapter.

[37] Oliver 2010: 127.

[38] It could be answered that the identity does not obtain after all, that is, that if F and G are predicates, F occurs in a language L_1 in which it is not construed as an expression, and G occurs in L_2 where it is so construed, then F and G cannot be the same predicate. I suspect that that manoeuvre would be question begging. In any case, it is not Oliver's.

1 Are there alternative analyses?

I approach the question of whether there are alternative analyses from the point of view of Ramsey's problem, that is, of whether there *may* be alternative analyses. I explain what is it about linguistic functions that makes them fit to dispel Ramsey's concern.

However, I agree with Dummett that analysis is unique in his sense. I reject the typical objections to Dummett's account, and argue, with Peter Sullivan, that Geach's functional model can not only accommodate, but actually imply Dummett's distinction between analysis and decomposition. In any case, it is again the notion of a linguistic function that explains why there may be alternative decompositions.

1.1 Ramsey's problem

It is worth quoting in full an observation of Ramsey's about the naïve theory of universals according to which each constituent of a sentence stands for a certain extra-linguistic entity: proper names for particulars, predicates for universals.

> In order to make things clearer let us take a simpler case, a proposition of the form '*aRb*'; then this theory will hold that there are three closely related propositions; one asserts that the relation *R* holds between the terms *a* and *b*, the second asserts the possession by *a* of the complex property of "having *R* to *b*", while the third asserts that *b* has the complex property that *a* has *R* to it. *These must be three different propositions because they have different sets of constituents, and yet they are not three propositions, but one proposition, for they all say the same thing, namely that a has R to b.* So the theory of complex universals is responsible for an incomprehensible trinity, as senseless as that of theology.[39]

Ramsey's problem is thus that of conceiving how it is possible for a single sentence to have alternative analyses, each showing that it says different things. Or, what is the same thing, how can a single sentence be correlated with alternative sets of constituents.

In a rather synthetic passage, Geach remarks that

> If a proposition were analysable into a predicate and one or more subjects – these being actual expressions, and constituents of the proposition – then one might well expect that […] the analysis will be unique.[40]

Now, what counts as a constituent is relative to the kind of analysis in question.

A subject-predicate analysis of '*aRb*' would identify '*a*' as subject and '*Rb*' as predicate. This would be the kind of representation demanded by the, by Frege's time traditional, Aristotelian syllogistic. Any sentence that occurs in a syllogism must be of the subject-predicate form, and include a quantifier and a copula. If a sentence such as 'Cato killed Brutus' were to occur in a syllogism, it would have to be artificially regimented for instance as 'Cato is the killer of Brutus'. The predicate 'the killer of Brutus' would be treated as a single term, so that its logical complexity could not be recognized. 'Something is the killer of Brutus' could perhaps be said to follow from 'Cato is the killer of Brutus', but not 'Cato is the killer of someone'. As Michael Potter has said, in syllogistic the logical connection between a killer and its victim cannot be

[39] Ramsey 1925: 405-6. My italics.
[40] Geach 1975: 146.

recognized.[41] Subject-predicate analysis is therefore both incredibly restrictive and many a time artificial.[42]

A relational analysis of '*aRb*' would identify as its constituents the *relata* '*a*' and '*b*' and the relation-word '*R*'. This basically corresponds to the grammatical analysis of a sentence of this form, which would in general identify as grammatical constituents a nominative, a verb, and an accusative: '*a*', '*R*' and '*b*', respectively.

Now '*aRb*' can certainly be analysed in either of these alternative ways: but each method assigns a unique set of constituents to the sentence. The subject-predicate account does not recognize the relational predicate '$\xi R\zeta$', and the relational account recognizes neither of the monadic predicates '$aR\xi$' nor 'ξRb'.

Geach sometimes seems to imply that the difference between grammatical and logical predicates is that only the former have typographical criteria of identity, or that, which amounts to little more than that, that only the latter are linguistic functions. But this cannot be right: if grammatical predicates are predicates at all, they must be linguistic functions in Geach's sense.[43] Perhaps what Geach means is merely that grammatical predicates were traditionally conceived as typographically identifiable.

The difference between traditional predicates and Fregean function-names is that function-names are more general: indeed as general as possible.[44] Traditional and grammatical predicates are only particular cases of linguistic functions, whereas function-names are just linguistic functions.

This is why Frege's singular method of analysis in terms of argument and function has no difficulty recognizing a sentence as instantiating more than one pattern. In the subject-predicate account, the sentence '*aRb*' is the value of the function 'ξRb' for '*a*' as argument, and of no other; the relational account represents '*aRb*' as the value of the function '$\xi R\zeta$' for '*a*' and '*b*' as arguments, and of no other. Fregean analysis is able to represent a single sentence as the value of different functions: not only of 'ξRb' and '$\xi R\zeta$', but also of '$aR\xi$' for '*b*' as argument, of '$a\Phi b$' for '$\xi R\zeta$' as argument, etc.

Frege's method is more powerful, then, because it explores the full generality of the notion of a linguistic function. As a consequence it is able to identify every possible pattern according to which a sentence may be constructed. A sentence such as '*aRb*' may be conceived as having been constructed from the concatenation of either '*a*' with '*Rb*', '*aR*' with '*b*', '*a*' with '*R*' and then with '*b*', etc.

As Geach says, the same thing may be the value of different functions.[45] Geach illustrates the idea with a simple example from arithmetic. 16 may be the value of the function ξ^2 for the argument 4, but it may also be the value of the function 2ξ for the argument 8. There is no mystery at all in the fact that the same number may be represented as the value of (indefinitely many) different functions, because just that is part of our understanding of what the relation between a function and its value for some argument amounts to. The relation between value and function is one-many. So if we conceive a predicate as a linguistic function, we should have no trouble understanding why a single sentence may be the value of different linguistic functions.

Now Geach says that

[41] Potter 2000: 64.
[42] Geach 1961: 151.
[43] See chapter 2.
[44] Geach 1961: 151.
[45] Geach 1975: 146.

> If we indeed could not analyse one and the same proposition in different ways, then, as Frege remarked, logic would simply be crippled.[46]

Correctly so, because it is the fact that '*aRb*' may be alternatively analysed in Frege's way that allows us to recognize that it shares common patterns with '*aRa*', '*bRb*', '*bRa*', '*aFa*', etc, so that the logical connections between each of them can be represented in a perspicuous manner.

Thus Geach remarks that

> Ramsey's difficulty is only specious. The resolution of the difficulty points up the advantage of Frege's apparatus over the old subject-predicate account.[47]

In general, to analyse a sentence is to show that it follows a pattern of construction shared by other sentences. A constituent of a sentence just is one of the items used in its construction. Earlier accounts recognized only one pattern of construction: the concatenation of a name with a concept-word, and the concatenation of a relation-word with its relata. Geach's notion of a linguistic function generalizes the notion of a pattern of construction to encompass any legitimate formation rule. Construing Frege's function signs as linguistic functions allows us to explain why is it that a sentence may be alternatively analysed in Frege's way. That different predicates may be extracted from a single sentence just means that it may be conceived as having been constructed in different ways. Each sentence may be correlated with alternative sets of constituents because different items are used in each such construction. It is therefore the greater generality of the notion of a function name, framed in terms of the notion of a linguistic function, which explains the intelligibility of alternatively analysing sentences.

1.2 Dummett on analysis and decomposition

There must be, however, something amiss with Geach's description of what would have been Frege's solution to Ramsey's problem.

According to Geach, to analyse a sentence is to represent it as the value of a linguistic function. But Geach takes this to be equivalent to exhibiting it as instantiating some one of the patterns obtained by predicate extraction.

Now the function-name '$\xi R\zeta$' does occur in '*aRb*': it is indeed one of the patterns discernible within the sentence. But '$\xi R\zeta$' has unrelated argument-places. As we have seen, for Frege, such predicates can only be extracted from expressions which already have "replaceable parts". That is, the relation-word '$\xi R\zeta$' cannot be extracted from '*aRb*', but only from either of the predicates 'ξRb' or '$aR\xi$'. It would seem, then, that '*aRb*' is the value of a linguistic function that cannot be immediately extracted from it. Therefore, not every pattern that a sentence may instantiate corresponds to a function-argument analysis of the sentence: some such patterns must correspond to function-argument analysis of other predicates that occur in the same sentence.

Ay, there's the rub. If to analyse a sentence is to exhibit it as a value for some pattern that it may instantiate, then it cannot be just a matter of applying Frege's method to it. Contrapositively, if to analyse a sentence is just to analyse *it* in terms of predicate

[46] Geach 1975: 146.
[47] Geach 1975: 146.

extraction, then it cannot be to exhibit it as instantiating just any possible pattern that it does instantiate.

In other words, most sentences are the values of more linguistic functions than those that can be extracted from them. Geach's general conception of analysis cannot therefore be entirely right; it must at best be incomplete.

This is at bottom the essence of Dummett's critique of Geach's account. Ironically, Dummett could not have presented it in such straightforward terms. Dummett writes:

> An exact account of decomposition is arrived at as follows. […] The simplest type of decomposition is into a proper name and a (one-place) predicate or functor. This is effected by the removal from the sentence or complex term of one or more (not necessarily all) occurrences of a proper name […]. The most immediate generalization is to the case when the incomplete expression that is left has two or more argument-places, and so forms an *n*-ary relational expression or functor. When *n*=2, we have two cases. The first is that in which we remove one or more occurrences of each of two distinct proper names […]. The second case is that in which we remove, say, *m* occurrences of some one proper name, regarding *k* of them as to be filled by a proper name, the same for each of them, and the remaining $m - k$ by another proper name, perhaps a distinct one.[48]

Dummett is right to observe that if we overlook the second case, "we shall have disallowed the legitimate decomposition of 'Brutus killed Brutus' into 'Brutus', 'ξ killed ζ' and 'Brutus'." What he fails to notice is what both Geach and Bermúdez also had. The latter is no single decomposition. There is no "generalization" of decomposition: there is only the "simplest type". Predicate extraction applies to both possible and "impossible" contents of judgement, i.e. to proper- and function-names alike. The two cases distinguished by Dummett in his generalization of decomposition are, as we have seen, described by Frege as two distinct applications of the operation of predicate extraction. Again, relation-words with unrelated argument-places can only be extracted from concept-words.

It is nevertheless enlightening to consider how the objection may be presented in Dummett's terms. Dummett's distinctions between simple and complex predicates, constituents and components, and analysis and decomposition, may indeed be fruitfully adopted. My exposition of Dummett's account is, however, reconstructive at critical points.

1.2.1 Simple and complex predicates

A predicate occurs in a sentence if it can be extracted from it in the sense of *Bs*, or if it can be extracted from a predicate extracted from it, and so on. A simple predicate is a predicate whose occurrence in a sentence implies the occurrence of no other determinate predicate but itself. A complex predicate is a predicate whose occurrence implies the occurrence of at least another determinate predicate. I restrict these definitions to predicates of the same level.[49]

Thus 'ξ killed Brutus' is a complex predicate since any sentence in which it occurs is a sentence in which the relation-word 'ξ killed ζ' also occurs, as the latter can be extracted from the former. The predicate 'ξ killed ξ' is also complex since any

[48] Dummett 1981: 274-5.
[49] The fruitfulness of these definitions will eventually show itself.

sentence in which it occurs is a sentence in which 'ξ killed ζ' also occurs. For instance, 'Brutus killed Brutus' mentions both 'ξ killed ξ' and 'ξ killed Brutus', and again 'ξ killed ζ' can be extracted from the latter. A sentence that expresses the concept of suicide thereby expresses the concept of murder, though not conversely. 'ξ killed ζ' is simple because although any sentence in which it occurs will involve some complex predicate, we cannot tell in advance which. In general, relation-words with related argument-places are complex predicates are relation-words with unrelated argument-places are simple predicates.

Hence simple predicates define the expressive resources of a language, whereas complex predicates add nothing to them.[50] The distinction does not quite coincide with basic vs. defined non-logical vocabulary: 'ξ killed ξ' could be among the basic vocabulary in place of 'ξ killed ζ', but the latter would still be expressed by any sentence which involved the former. In any case, complex predicates are formed from sentences which result from combinations of simple predicates, names and operators.

1.2.2 Decomposition and analysis

Dummett terms Frege's procedure of function-argument analysis, or predicate extraction, decomposition.

In the simplest cases, a single decomposition of a sentence may be sufficient to uncover the simple predicates it involves. For instance, the simple predicate 'ξ is wise' can be extracted from 'Plato is wise'. But in the vast majority of cases, as Frege was well aware, it may be impossible to arrive at simple predicates by a single decomposition. The simplest such case would be that in which a sentence involves a relation-word with unrelated argument-places. The predicate 'ξ killed ζ' does occur in 'Brutus killed Caesar', but a decomposition of the latter yields only either one of the concept-words 'ξ killed Caesar' or 'Brutus killed ξ', which are both complex.

Decomposition is therefore generally inappropriate to uncover simple predicates. On the other hand, the identification of complex predicates is particularly useful for the discovery of fruitful concepts, and for exhibiting the validity of inference. Decomposition is thus primarily designed to aid concept-formation, and to explain the fruitfulness of deductive reasoning.[51]

In order to arrive at simple predicates it is many a time necessary to establish a series of successive decompositions, beginning with sentences, then moving to their subcomponents, which may be either sentences or complex predicates. Analysis is the series of decompositions which ends when only simple predicates are left.

At the last stage of analysis there can be found only names, simple predicates and operators. In turn, these are the items that are present at any stage of the construction of the sentence.[52] Thus analysis is the inverse process of construction. To analyse a sentence is to follow the order of its construction in reverse steps. The representation of a sentence in its fully analysed form reveals what Dummett calls (with no temporal connotation) its constructional history.[53]

Just as analysis has a constructive converse procedure, so does decomposition; we could call the converse of the latter 'saturation'. What distinguishes analysis and decomposition is therefore not the fact that one has a constructive counterpart while the

[50] Sullivan 2010: 108.
[51] Dummett 1981: 290-1.
[52] See next section.
[53] Dummett 1973: 34-5.

other does not, *pace* Dummett. The difference lies rather in that analysis essentially proceeds in stages, and it is only in virtue of this feature that it is able to fulfil what decomposition in most cases cannot.

Analysis is complete only when the stage is reached in which no complex predicates are left, whereas a single decomposition is exhausted at each stage of analysis. Conversely, construction is completed only after its last step; on the other hand, each step of construction consists in a single operation of saturation.

1.2.3 Components and constituents

If a predicate occurs in a sentence at all, it is one of its components. Components that occur in some step of the construction of a sentence are its constituents. Components that occur at every such step are the ultimate constituents of sentences.

Conversely, any predicate revealed at some stage of analysis is a constituent of the sentence. Constituents left at its last stage are ultimate. Only simple predicates may therefore be ultimate constituents. Complex predicates may be constituents but never ultimate, since they must "disappear" at later stages of analysis.

Note that to say that a predicate occurs at every step of the construction of a sentence is to say that it can be obtained at any such step by either analysis or decomposition.

Both 'ξ killed Caesar' and 'Brutus killed ξ' are constituents of 'Brutus killed Caesar'. The latter can be constructed from the saturation of the first predicate by 'Brutus' or from that of the second by 'Caesar'. Conversely, the extraction of either 'ξ killed Caesar' or 'Brutus killed ξ' corresponds to the first stage of the analysis of the sentence. But the first step of its construction involves only 'ξ killed ζ' and, say, 'Caesar'. So the complex predicate 'ξ killed Caesar' is not present at every step of construction; it is therefore not an ultimate constituent of the sentence. On the other hand the simple predicate 'ξ killed ζ' occurs in every such step, including the first, and so it is an ultimate constituent of 'Brutus killed Caesar'.

Since simple predicates must be ultimate constituents, they cannot be merely components. On the contrary, some complex predicates may be components without being constituents. The predicate 'ξ killed ξ' is a component but not a constituent of 'Brutus killed Brutus', since, although it may be extracted from the sentence, it appears at no stage of its analysis. Decomposition is never arbitrary. In particular, the decomposition that corresponds to the first stage of the analysis of a sentence must always be applied with an eye at uncovering simple predicates. 'ξ killed ζ' is also a component of 'Brutus killed Brutus'. Therefore the extraction of 'ξ killed ξ' cannot be the first – or indeed any – step of the analysis of the sentence, since it would preclude the extraction of 'ξ killed ζ'. And so 'ξ killed ξ' is a component, but not a constituent, let alone an ultimate constituent, of 'Brutus killed Brutus'.

Dummett is keen to show how his distinctions come out in the explanation of the structure of quantified sentences. Sentence (2) below follows from (1) by universal instantiation.

(1) If anyone killed Brutus, he was an honourable man.
(2) If Brutus killed Brutus, Brutus was an honourable man.

In order to recognize the validity of the inference it is useful to exhibit both sentences as instantiating a common pattern.

If anyone **killed Brutus,** he **was an honourable man.**
If Brutus **killed Brutus,** Brutus **was an honourable man**.

This amounts to showing that the complex predicate 'If ξ killed Brutus, ξ was an honourable man' is a component of both sentences.

Now, the last step of the construction of (1) is the universal closure of 'If ξ killed Brutus, ξ was an honourable man'. This means that the complex predicate is also a constituent of (1): it is required by the first stage of its analysis.

On the other hand, the analysis of (2) does not involve such decomposition. It rather relies on the identification of another pattern, 'If ξ, then ζ'. The last two steps of the construction of (2) are the saturation of the argument-places of the operator 'If ξ, then ζ' by 'Brutus killed Brutus' and 'Brutus was an honourable man' respectively. Thus the complex predicate 'If ξ killed Brutus, ξ was an honourable man' is a component but not a constituent of (2).

But the complex predicate is not an ultimate constituent of (1) either. The next stage of the analysis of (1) would involve recognizing that 'If ξ killed Brutus, ξ was an honourable man' had itself been extracted from a sentence such as (2). Note that the complex predicate could not have been formed from the saturation of 'If ξ, then ζ' by 'ξ killed Brutus' and 'ξ was an honourable man' since 'If ξ, then ζ' takes only complete expressions (in this case sentences) as arguments. From the third stage on, the analysis of (1) would thus be similar to the analysis of any of its instances. Once (1) were completely analysed, we could see that it had been constructed from the simple predicates 'ξ killed ζ', 'ξ was honourable' and 'ξ was a man', the name 'Brutus', the propositional connectives 'If ξ, then ζ', 'ξ and ζ' and the universal quantifier.

1.2.4 The uniqueness of analysis

Our initial objection to Geach's account of Ramsey's problem may be stated in Dummettian terms thus: not every component of a sentence is available by a single decomposition; in particular, most ultimate constituents are not.

Geach was right to include '$\xi R\zeta$' among the components of '*aRb*', as the former occurs in the latter. He was also right in that it is always possible to analyse a sentence in the sense of *Bs* in more than one way, i.e. that decomposition is never unique.

Where Geach erred was when he implied that '$\xi R\zeta$' could be extracted from '*aRb*'. Not every component may be immediately extracted from a sentence. In particular, relation-words with unrelated argument-places cannot. We can only arrive at '$\xi R\zeta$' via an analysis of '*aRb*' in two stages, the first of which consists in the decomposition of '*aRb*' into, say, 'ξ*Rb*', and the second in the extraction of '$\xi R\zeta$' from the complex predicate.

In general, to arrive at simple predicates, we need analysis rather than decomposition. That is, we need to apply a series of successive decompositions, first to sentences, then to complex predicates, until only simple predicates are left. The simplest such case is that in which a sentence involves a relation-word with unrelated argument-places.

Now, any component is either a constituent, or it is not. If it is not, *a fortiori* it is not an ultimate constituent. If it is, then it is either a simple or a complex predicate. If a constituent is a complex predicate, then it can be constructed from ultimate constituents, and so is not among ultimate constituents. If a constituent is a simple predicate, then by

definition it implies the occurrence of no other determinate predicate. And so for each sentence, there may be only one formula that represents its constructional history.

In general, unlike decomposition, analysis is unique because the ultimate constituents of a sentence must always form a unique set.

Another way to state that analysis is unique is to say that decomposition presupposes analysis. That is to say that (i) any complex predicate that constitutes a sentence can be constructed from the simple predicates that ultimately constitute the sentence; and (ii) any complex predicate that merely composes a sentence implies the occurrence of some simple predicate that ultimately constitutes it.

In other words, analysis, or the set of ultimate constituents of a sentence, circumscribes all the possible decompositions that a sentence may legitimately be subject to, and all the possible decompositions that its components may be subject to. In turn, this is only an alternative way of expressing that predicate extraction is constrained by the smallest significant idioms of sentences.

However, Dummett seems to have underestimated the extent to which analysis also implies decomposition. He focused on the fact that the analysis of quantified sentences necessarily involved the recognition of a complex predicate. But in fact each stage of analysis consists in a decomposition of some sentence or complex predicate: the final stage, indeed, in a decomposition from which result only simple predicates. Dummett was strangely led to the claim that every predicate arrived at by decomposition should be complex; contrapositively, that decomposition did not apply to simple predicates. Dummett developed what, from a Fregean point of view, is quite an unorthodox doctrine of simple predicates.

1.2.5 Dummett's doctrine of simple predicates

The fundamental distinction between simple and complex predicates is that all and only simple predicates can be among the ultimate constituents of sentences. But Dummett introduces a further distinction between the two classes.

As Geach put it, Dummett's further distinction "is not a matter of number of words, but of quotability."[54] Like a proper name and unlike a complex predicate, a simple predicate "constitutes a word that is physically capable of being detached from the sentence."[55] In a word, simple predicates do not belong to the category of incomplete expressions.

> Simple predicates are *selbständig* in the way that complex ones are not: they are merely words or strings of words which can quite straightforwardly be written down. In one sense, of course, they are incomplete – they do not constitute a sentence, a "complete utterance": but, in that sense, proper names are equally incomplete.[56]

On the other hand, it is complex predicates that "form the prototype for Frege's general notion of an 'incomplete' expression:"

> Such expressions are said by him to contain gaps, and, further, to be *unselbständig*: they cannot subsist – they cannot stand up, one might say – on their own. If one considers complex predicates formed by the omission of more than one occurrence of the same proper name from a sentence, the purport of this is immediately clear.[57]

[54] Geach 1975: 147.
[55] Dummett 1973: 28.
[56] Dummett 1973: 32.
[57] Dummett 1973: 31.

Complex predicates are not mere sequences of signs occurring in sentences, but rather sequences "standing in a certain uniform relation to terms occurring in those sentences." They are recognized as a common feature of the various sentences in which they occur. That is what explains that they are not quotable bits of language, and can only be indicated in isolation by the use of Greek letters, which represent how they are related to other terms in the sentences from which they are extracted. In turn, this is why we are compelled to regard a complex predicate as formed from a sentence rather than as built up from its components.[58]

We can sum up the difference between complex and simple predicates thus: simple and complex predicates alike have relational properties (determined by their grammar), but only the latter have them essentially. Simple predicates can be straightforwardly identifiable by typographical criteria alone, since the relations they bear to other terms are not essential to them; but complex predicates are essentially incomplete expressions. Thus Dummett writes that

> We might say that, in the case of simple predicates, the slots are external to them, whereas in the case of complex predicates, they are internal. That is, we can know what linguistic entity, considered just as a sequence of phonemes or of printed letters, a simple predicate is, without knowing anything about the slot it carries with it: the slot consists merely in the predicate's being subject to a certain rule about how it can be put together with a term to form a sentence. But the complex predicate cannot be so much as recognized unless we know what slots it carries: they are integral to its very being.[59]

Dummett's insistence that only complex predicates are incomplete expressions proper leads him to a sort of paradox. Since he ties incompleteness to decomposition, and decomposition to complex predicates, he is led to believe that there is no sense to attach to a simple predicate's being extracted from a sentence, and, by the same token, no way in which a complex predicate may be obtained if not by predicate extraction. A "complex predicate is formed, not directly out of its constituent expressions, but from a sentence in which it occurs."[60]

Now, the predicate 'ξ snores' *can* be extracted from a sentence such as 'Herbert snores', by the omission of the name 'Herbert'. But then it must be a complex predicate; for Dummett, indeed a degenerate one. Let '… snores' be the simple predicate from which 'Herbert snores' is built. Then a certain redundancy occurs: 'Herbert snores' involves both 'ξ snores' and '… snores'.

> Nevertheless, it remains the case that, strictly speaking, if 'ξ snores' is treated as a complex predicate, on all fours with, say, 'If anyone snores, then ξ snores', we do need to recognize the separate existence of the simple predicate '... snores' as well: for, precisely *because the "complex" predicate 'ξ snores' has to be regarded as formed from such a sentence as 'Herbert snores', it cannot itself be one of the ingredients from which 'Herbert snores' was formed*, and thus cannot be that whose sense, on Frege's own account, contributes to composing the sense of 'Herbert snores'.[61]

Dummett concludes somewhat arbitrarily that if there is to be any economy, it is the complex predicate 'ξ snores' that should be dispensed with, since 'Everyone snores' may be regarded as a special case in which a quantifier is attached to the simple

[58] Dummett 1973: 31.
[59] Dummett 1973: 32-33.
[60] Dummett 1973: 30.
[61] Dummett 1973: 30-1. My italics.

predicate '… snores'. A similar problem would also arise as regards not decomposition but saturation, for any complex predicate that could be conceived as having been constructed from simple predicates.

Geach does not hesitate to condemn this doctrine of Dummett's as both "false and unFregean".[62] Indeed, for Geach as for Frege, every predicate would be complex in this new sense. One thing is to suppose that simple predicates are more easily recognizable, another to think that their "relational properties" are "extrinsic" to them. Dummett's view entails that the predicate in 'John killed Mary' is the bare word 'killed'. But it is only the relation that obtains between 'killed' and 'John' and 'Mary' in 'John killed Mary' that allows the latter to be distinguished from 'Mary killed John'. If the relation were not essential to the predicate, the sentences would be indistinguishable: but they aren't, and so it is. The expression 'ξ killed ζ' codifies the linguistic relation, or rule: The name ξ is written just before, and the name ζ just after some token of the word 'killed'. "The point is so obvious and so trite as not to need illustration."[63] It is essential to the predicate that it involves its formation rule, even though the latter might not be given by word order.

Even someone like Alex Oliver, for whom the fact that predicates involved essentially their formation rules would not turn them into linguistic functions, claims that Dummett's simple predicates cannot be expressions *simpliciter*. It is true that 'killed' occurs in 'Brutus killed Brutus' and 'Brutus killed Caesar', as does 'ξ killed ζ', whereas 'ξ killed ξ' occurs only in the former sentence despite the fact that both have all parts in common.[64] But it does not follow that 'ξ killed ζ' just is the bare word 'killed'. What distinguishes 'ξ killed ξ' from 'ξ killed ζ' is precisely that the argument-places of the first must be filled by the same proper name, while those of the second may be filled by the same or by different ones. The fact that complex and simple predicates serve different explanatory roles does not entail that they have different natures.[65]

Prima facie, simple and complex predicates alike can be both extracted from sentences and construct them. In fact, as we have seen, for a predicate to be extracted from a sentence just means for Geach that the sentence may be conceived as having been constructed through it. Dummett seems to have inexplicably neglected that decomposition has as much a constructive converse procedure as analysis. I take it that this may have been one of the motives, or one of the consequences, of Dummett's unfortunate choice of presenting his case against Geach's account of analysis in a way which may have in the end caused some harm along with some good.

1.2.6 The A and the B theses

Dummett finds a conflict between two sets of doctrines which he accurately attributed to Frege:

A1 A thought may be analysed in different ways.
A2 A thought is not built up out of its component concepts; rather, the constituents of the thought are arrived at by analysis of it.

[62] Geach 1976a: 445.
[63] Geach 1975: 148.
[64] Dummett 1973: 31.
[65] Oliver 2010: 125-6.

B1 The senses of the parts of a sentence are parts of the thought expressed by the whole.

B2 A thought is built up out of its constituents, which correspond, by and large, to the parts of the sentence expressing it.[66]

According to Dummett, each set of theses exemplifies a particular conception of the relation that a thought, or a sentence, may bear to its parts. The A theses correspond to a decompositional model of the relation between sentences and their constituents, while the B theses correspond to a compositional or constructional one.

Frege did state the A and the B theses, both in print and in private correspondence, over a significantly overlapping period of time. Dummett is right to observe that the A theses are characteristic of his earlier period. As early as 1882, in a letter to colleague Anton Marty, Frege wrote:

> Now I do not believe that concept formation can precede judgement because this would presuppose the independent existence of concepts, but I think of a concept as having arisen by decomposition from a judgeable content. I do not believe that for any judgeable content there is only one way in which it can be decomposed, or that one of these possible ways can always claim objective pre-eminence.[67]

The A theses survive the bifurcation of the notion of conceptual content into those of sense and reference.

> […] a thought can be split up in many ways, so that now one thing, now another, appears as subject or predicate.[68]

> We must notice, however, that one and the same thought can be split up in different ways and so can be seen as put together out of parts in different ways.[69]

> So I do not begin with concepts and put them together to form a thought or judgement; I come by the parts of a thought by analysing the thought.[70]

On the other hand, the B theses are typical of Frege's more mature period.

> It is remarkable what language can achieve. With a few sounds and combinations of sounds it is capable of expressing a huge number of thoughts, and, in particular, thoughts which have not hitherto been grasped or expressed by any man. How can it achieve so much? By virtue of the fact that thoughts have parts out of which they are built up. And these parts, these building blocks, correspond to groups of sounds, out of which the sentence expressing the thought is built up, so that the construction of the sentence out of parts of a sentence corresponds to the construction of a thought out of parts of a thought. And as we take a thought to be the sense of a sentence, so we may call a part of a thought the sense of that part of the sentence which corresponds to it.[71]

The same view would be repeated almost *ipsis verbis* in "Compound Thoughts":

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[66] Dummett 1981: 361.
[67] Frege 1882: 101.
[68] Frege 1892b: 199.
[69] Frege 1906c: 201-2.
[70] Frege 1919: 253.
[71] Frege 1914a: 225.

> It is astonishing what language can do. With a few syllables it can express an incalculable number of thoughts, so that even if a thought has been grasped by an inhabitant of the Earth for the very first time, a form of words can be found in which it will be understood by someone else to whom it is entirely new. This would not be possible, if we could not distinguish parts in the thought corresponding to the parts of a sentence, so that the structure of the sentence can serve as a picture of the structure of the thought.[72]

And in a letter to Jourdain from 1914, Frege had also written that:

> The possibility of our understanding propositions which we have never heard before rests evidently on this, that we construct the sense of a proposition out of parts that correspond to the words. If we find the same word in two propositions, e.g., 'Etna', then we also recognize something common to the corresponding thoughts, something corresponding to this word. Without this, language in the proper sense would be impossible.[73]

Dummett reconstructs Geach's account of Ramsey's problem as basically consisting in the rejection of the B theses. On the one hand, Geach did deny that analysis were unique; on the other Geach argued that Frege's use of the part-whole vocabulary was a mere metaphor.[74] Dummett points out that even if Geach's account were a "satisfactorily Fregean" solution to Ramsey's difficulty, it would not be a satisfactory solution of the exegetical problem of the apparent conflict between the A and the B theses, since it simply jettisons the latter. According to Dummett, Geach's view would entail ascribing to Frege a sufficient amount of confusion for him to fail to recognize "a plain contradiction when it stared him in the face".[75]

Dummett solves the apparent conflict by claiming that each model relates to a different kind of analysis. He associates the A theses with decomposition and the B theses with analysis. Since decomposition is never unique, it satisfies A1; since it aims at discovering components, it satisfies A2. On the other hand, analysis aims at uncovering ultimate constituents, which is to say that it is the converse of construction, and so satisfies B2. Ironically, Dummett himself seems to have been aware that, while the A theses apparently imply that analysis is not unique, the B theses do *not* imply that analysis is unique. Rather, the compositional model only "suggests that the thought is capable of a *unique* analysis into its ultimate constituents."[76]

The reason, of course, is that there is no contradiction between the A and the B theses at all. As Dejnožka put it, "composition and decomposition are distinct only in reason".[77] Dejnožka actually cites a remarkable passage of Frege's to this effect.

> The mental activities leading to the formulation of a definition may be of two kinds: analytic or synthetic. This is similar to the activities of the chemist, who either analyses a given substance into its elements or lets given elements combine to form a new substance. In both cases, we come to know the composition of a substance.[78]

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[72] Frege 1923: 36.
[73] Frege 1914b: 79.
[74] See chapter 3.
[75] Dummett: 1981: 264, 266.
[76] Dummett 1981: 263.
[77] Dejnožka 2007: 88.
[78] Frege 1906a: 303.

So while it is true that Geach rejected the terminology of the B theses, he would have been right to claim, merely on the basis of the A and the B theses, that analysis is not unique, even if there were some contradiction as Dummett contended. The uniqueness of analysis has therefore nothing to do with the A and the B theses.

It seems extraordinary that Dummett did not see that decomposition and saturation are the converse of each other, just as analysis is the converse of construction. Thus decomposition is as much committed to the B theses as analysis. Besides, since the A theses relate to predicate extraction, as Dummett notes correctly, both sets of doctrines could be accommodated solely by decomposition without any threat of inconsistency.

The key to sorting out Dummett's confusion lies, I believe, in his interpretation of the principle of the priority of judgement over concepts, expressed by A2. Dummett gives the principle a psychologistic reading, so to say.

Dummett denied that the priority of judgement applied to simple predicates solely on the ground that they are the ultimate constituents of sentences.[79] If simple predicates are among ultimate constituents, they are necessary and sufficient for understanding a sentence. But how could that be if they did not precede the sentences they construct?

The priority of judgement has indeed been given ontological and epistemic readings. But in essence, it is as much a methodological principle as Frege's context principle: never to ask for the meaning of a word in isolation, but only in the context of a sentence.[80]

My diagnosis is that Dummett was here oblivious of his own salutary distinction between the order of recognition and the order of explanation.

> Frege's account, if it is to be reduced to a slogan, could be expressed in this way: that in the order of explanation the sense of a sentence is primary, but in the order of recognition the sense of a word is primary. Frege was unwaveringly insistent that the sense of a sentence – or of any complex expression – is made up out of the senses of its constituent words. […] But, when we come to give any general explanation of what it is for sentences and words to have a sense, that is, of what it is for us to grasp their sense, then the order of priority is reversed.[81]

This distinction allowed Dummett to hold on to the context principle for simple and complex predicates alike. Oddly enough, he explicitly distinguished the priority of judgment from the context principle precisely because the former could not apply to simple predicates. Why he did so is to me as yet baffling; the reason may be related to an ambiguity in what Dummett meant by linguistic understanding. For this reason, I have deliberately abstained from framing Dummett's account of analysis in terms of sense. This approach is authorized by Dummett himself, who stresses that it can be explained without invoking the notion of sense.[82] In any case, this is not the place to discuss Dummett's theory of understanding.

An immediate consequence of the rejection of the priority of judgement for simple predicates is that they cannot be obtained by predicate extraction, which Dummett equates with that principle. This is the origin of the paradoxical reasoning that Dummett had followed to conclude that 'Herbert snores' should include both a simple

[79] Dummett 1981: 281.
[80] Frege 1884: xxii. On a similar point, see Sullivan 2004: 688.
[81] Dummett 1973: 4.
[82] Dummett 1981: 272.

predicate and a degenerate complex one. There is no need for such a multiplication of entities: the sentence includes only a simple predicate, because simple predicates too can be extracted from sentences.

In the end, there is no reason to suppose that simple predicates are "less incomplete" then complex ones. Not only do simple predicates involve their formation rules essentially: they are identified as patterns within sentences just as well.

By the same token, since decomposition is but the converse of saturation, not only can simple predicates be saturated by names to form sentences, but so can complex ones. It is thus not true in general that complex predicates cannot be conceived as serving to construct sentences, neither that their incompleteness is tied to the fact that they must be regarded as extractable from sentences.

However, nothing of the above counts against Dummett's explanation of the difference between analysis and decomposition, which relies on the distinction between constituents and components. What is essential is that analysis proceeds in stages – indeed, in stages of successive decompositions, first of sentences, then of complex predicates, until only simple predicates are left. It is beguiling that Dummett could not have stated matters in this way, since, for him, simple predicates could not be obtained by decomposition.

Gregory Currie was therefore completely off when he mounted a critique of Dummett's account of the uniqueness of analysis on the basis of the latter's muddied doctrine of simple predicates, although Dummett is certainly to blame for inviting the confusion.[83]

Currie finds a ground to ascribe to Frege the notion of a simple predicate as a predicate not further analysable in the following passage:

> And so instead of putting a judgement together out of an individual as subject and an already previously formed concept as predicate, we do the opposite and arrive at a concept by splitting up the content of possible judgement. Of course, if the expression of the content of possible judgement is to be analysable in this way, it must already be itself articulated. We may infer from this that at least the properties and relations which are not further analysable must have their own simple designations.[84]

Currie goes on to assert, quite rightly of course, that Dummett's conception of simple predicates as self-standing expressions is nowhere to be found in Frege. But ironically, the above is perhaps the best text in which to look for evidence that Frege thought that analysis were unique in Dummett's sense. What Frege implies here is basically that decomposition presupposes analysis: the possibility of "splitting up the content of possible judgement" presupposes that the content is "already itself articulated". The permissible decompositions of a sentence, and the permissible decompositions of any of the complex predicates that occur in it, are circumscribed by its structure, i.e. its constructional history as exhibited by analysis. The properties and relations "not further analysable", i.e. the ultimate constituents, "must have their own simple designations", i.e. be simple predicates.

[83] Currie 1985: 285-7.
[84] Frege 1881: 17.

1.3 Objections to Dummett's interpretation

Objections to Dummett's interpretation come in two kinds. The first concerns the rejection of Dummett's textual grounds for attributing his account of analysis to Frege. The second consists in presenting counter-examples to the uniqueness of analysis, that is, instances of sentences about which Frege would not have said that they had unique analyses.

1.3.1 Textual evidence

According to Dummett, the distinction between components and constituents was already clearly implicit in *Bs*.[85]

> We attach no importance to the various ways that the same conceptual content may be regarded as a function of this or that argument, so long as function and argument are completely determinate. But if the argument becomes *indeterminate*, as in the judgement: 'whatever arbitrary positive integer you may take as argument for "being representable as the sum of four squares", the proposition always remains true', then the distinction between function and argument becomes significant as regards the *content*. Conversely, the argument may be determinate and the function indeterminate. In both cases, in view of the contrast *determinate-indeterminate* or *more and less determinate*, the whole proposition splits up into *function* and *argument* as regards its own content, not just as regards our way of looking at it.[86]

The "conceptual contents" whose arguments are "completely determinate" are the sentences which do not involve bound variables, i.e. that do not express generality. The ways in which such sentences split up into argument and function regard only our way of looking at it. Dummett would interpret this as meaning that any complex predicate arrived at by decomposition of a non-quantified sentence is a component, not a constituent of the sentence. Dummett glosses a complex predicate's not concerning a sentence's own content as its being a mere component of the sentence: its identification is not necessary during the construction of the sentence, and so it does not appear in its analysis. And indeed the complex predicate 'If ξ is a positive integer, then ξ can be represented as the sum of four squares' occurs in 'If 20 is a positive integer, then it can be represented as the sum of four squares', but not as its constituent.

On the other hand, decompositions of quantified sentences – those which involve bound variables, i.e. arguments which become "indeterminate" – do regard their own content. This would mean, according to Dummett, that complex predicates that result from decompositions of quantified sentences are essential to their analysis, and to their construction. Therefore, such decompositions do regard the content of quantified sentences, and not merely our way of looking at them. The complex predicate 'If ξ is a positive integer, then ξ can be represented as the sum of four squares' occurs in 'Every positive integer can be represented as the sum of four squares' as its constituent.

Currie has argued that, although Dummett's reading is consistent with Frege's text, it ignores the context of the passage.[87]

The quotation above immediately follows a warning of Frege's against "a fallacy that ordinary language easily leads to": that of adhering too closely to its

[85] Dummett 1981: 281-2.
[86] Frege 1879 §9.
[87] Currie 1985: 288-9.

grammar for the purpose of logical analysis. Frege suggests a comparison between two sentences, 'The number 20 can be represented as the sum of four squares' and 'Every positive integer can be represented as the sum of four squares'. The first sentence can be decomposed into the argument 'The number 20' and the function 'ξ can be represented as the sum of four squares'. Due to the grammatical similarity between both sentences, one could be led to decompose the second sentence into the argument 'Every positive integer' and the function 'ξ can be represented as the sum of four squares'. Alas, the analogy is misleading.

> We may see that this view is mistaken if we observe that 'the number 2' and 'every positive integer' are not concepts of the same rank. What is asserted of the number 20 cannot be asserted in the same sense of [the concept] 'every positive integer'; of course it may in certain circumstances be assertible of every positive integer. The expression 'every positive integer' just by itself, unlike 'the number 20', gives no complete idea; it gets a sense only through the context of the sentence.[88]

Currie thus reads the passage as entailing only that the grammatical subjects of quantified sentences are not singular terms. The reason that Frege does not attach particular importance to the alternative decomposition of a non-quantified sentence is that the content of such sentences will be such that any apparently permissible decomposition will in fact be permissible. On the other hand, in quantified sentences, our regarding one part or another as the function or as the argument is constrained by the content of the sentence being such that some parts of it cannot be regarded as singular terms, or logical subjects. Currie concludes that in the light of such a simple explanation, Dummett's reading seems "overly ingenious".

However, Currie's and Dummett's readings are related in an obvious way. It is precisely because 'Every ξ' is not a proper name that the predicate to which it attaches, simple or complex, must be a constituent rather than a component. It is not clear, then, that both readings constitute a real alternative after all.

In any case, there is a further problem with Dummett's reading. Dummett's account applies equally well to sentences which do not involve generality. But Currie had already noticed that Frege does not make the same point concerning 'Cato killed Cato'.[89] As Sullivan notes, the passage implies that "Frege never himself applied the notion of an intrinsic or *inhaltlich* articulation to elementary propositions."[90] The implication would be that the distinction between components and constituents did not apply to them.

There is actually a way to rescue Dummett's interpretation. There is another sense in which the analyses of elementary and quantified sentences differ. The analysis of a sentence such as 'Cato killed Cato' may begin by one of two decompositions. But there is no alternative in the case of quantified sentences, which must always begin with a determinate decomposition. So perhaps the contrast between elementary and quantified sentences is not that the distinction between components and constituents that does not apply to the former, but rather that there may be alternative ways to begin the analysis of an elementary relational sentence.

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[88] Frege 1879 §9.

[89] Currie 1985: 288. Currie expresses himself in terms of complex and simple predicates, but that is a mistake. The predicate 'ξ can be represented as the sum of four squares' is complex as a matter of definition. However, his mistake is prompted by Dummett's misguided denial of the applicability of the priority of judgement to simple predicates.

[90] Sullivan 1992, note 7.

Only the case of monadic simple predication fails to fit Dummett's description. The analysis of 'Socrates is wise' has a unique stage, and *a fortiori* a unique first step. It would still have to be argued, then, that the context restricts Frege's remarks to more complex cases.

In any case, the distinction between constituents and components follows from the distinctions between simple and complex predicates, and between analysis and decomposition. But as we have seen, there are good reasons for attributing to Frege the views that (i) simple predicates are not further analysable; (ii) to arrive at simple predicates we may have to proceed by stages of decompositions; and (iii) that predicate extraction presupposes the uniqueness of analytical structure. In the light of all this, only prejudice could make us think of Dummett's account as a revision rather than as an explanation of Frege's thought.

1.3.2 Counter-examples to the uniqueness of analysis

In order to support the view that "an ideal symbolic notation would have a unique form of expression for each thought",[91] Dummett cites an excerpt from a letter to Husserl, in which Frege states that equipollent, i.e. synonymous, sentences have but one transcription into his concept-script.

> All that would be needed would be a single standard proposition for each system of equipollent propositions, and any thought could be communicated by such a standard proposition.[92]

Currie rightly observes that this "is not sufficient to show that Frege wanted a language in which Thoughts are paired uniquely with sentences."[93] Again he invites Frege's reader to consider the context that surrounds the passage above. Frege is explaining to Husserl that not every aspect of the content of an expression should be taken into account in logic: in short, he is reasserting his distinction between sense and tone. Two equipollent sentences may still differ in tone, but they will nevertheless correspond to a single "standard proposition", or "sentence in normal form", since Frege's concept-script is insensitive to such differences.[94] Sentences with different tones but identical sense will be paired with a unique sentence of the concept-script; but it does not follow that each sentence of the concept-script will be paired uniquely with a thought.

This time, though, it has been Currie himself that missed the context of Dummett's quotation, for Dummett is in fact making precisely that very point. Dummett invokes the passage in order to remark, as Frege had, that differences of tone might be useful to indicate a preferred decomposition of a sentence that will in general have a unique transcription into the concept-script. Currie's critique is therefore unfair: Dummett is claiming that differences in tone are not an objection to the uniqueness of analysis, not that the irrelevance of tone to logic entails it.

Of course that if thought is linguistic sense, the "unique pairing" of sentences of the concept-script with thoughts follows from their unique pairing with synonymous

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[91] Dummett 1981: 279.

[92] Frege 1906d: 67. Dummett's translation reads: "One would need to have for every system of equipollent sentences only a single sentence in normal form (*Normalsatz*), and could communicate every thought by means of these sentences in normal form."

[93] Currie 1985: 294.

[94] For more on sense and tone, see section 1.3.3.3 below.

sentences. Let us say that thought is linguistic sense if and only if there is a bijection from the sentences of the concept-script to thoughts.

Currie avers that there are two kinds of cases that fall under Frege's principle that the same thought may be analysed in different ways. The first is that in which a single thought is expressed by different sentences of the concept-script. Currie states as an instance of this Frege's Basic Law V; any instance of logical abstraction (such as Frege's numerical equivalence) would do:

> The value-range of *f* is identical to the value-range of *g*.
> All and only *f*s are *g*s.

In symbols:

> ἐ *f* (ε) = ἀ *g* (α)
> (∀*x*) (*f* (*x*) ≡ *g* (*x*))

The second kind of cases corresponds to the alternative decompositions that a sentence may have in Dummett sense, i.e. that different predicates may be extracted from a sentence.[95]

Currie goes on to argue that Frege could not have intended thoughts to be paired uniquely with sentences of the concept-script due to the equipollence between 'The value-range of *f* is identical to the value-range of *g*' and 'All and only *f*s are *g*s'.

> There are also reasons for thinking that Frege would not seriously have wanted to extrude from even an ideal language all but one sentence for each Thought. Recall his claim that the two sides of Basic Law V have the same sense. No language would be ideal in which that principle could not be expressed.
> This concludes my argument for (a').[96]

(Currie's thesis (a') is "There is no warrant for assuming that Fregean analysis is unique, and hence no conflict between A1 and the B theses which the analysis/decomposition distinction could be used to resolve."[97])

It is true that it is not clear that Basic Law V and like principles pass any of Frege's criteria of synonymy. It is also true that probably Frege should *not* have claimed that the two sides of such principles were synonymous, on pain of the equivalences' loosing their explanatory power.[98] It is nonetheless clear that Frege would accept that different sentences of the concept-script could be mapped onto a single thought.

However, this only counts against Dummett's view that in general thought is linguistic sense. On the other hand, it does not even count against the view that, in most cases, thought *is* linguistic sense. It may just be that thoughts of a peculiar sort, namely those that involve reference to abstract entities, may be expressed by non-synonymous, differently structured sentences.

Possibly, that thought is linguistic sense is an unargued premise, or, better, a presupposition of Dummett's entire approach to Frege. That is fair enough; although Dummett's is obviously not an absurd hypothesis, given that Frege does state that a thought is the sense of a sentence.

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[95] Currie 1985: 290-1.
[96] Currie 1985: 294.
[97] Currie 1985: 292.
[98] See for instance Dummett 1991a: 168-76.

What does not follow, *pace* Currie, is that analysis is not unique, at least if by 'analysis' one means the procedure described by Dummett: the series of decompositions in Frege's sense of predicate extraction. The sentences 'The value-range of *f* is identical to the value-range of *g*' and 'All and only *f*s are *g*s' are *not* alternative analyses of the same thought, in this sense of analysis. If they arise from the same thought by "object-formation", to borrow Michael Beaney's term, rather than by "concept-formation", that is an altogether different procedure, that does not in the least correspond to the method of function-argument analysis as presented by Frege in *Bs* §9, i.e. Dummett's decomposition. Note that each side of an equivalence such as 'ἐ *f* (ε) = ἀ *g* (α) ≡ (∀*x*) (*f* (*x*) ≡ *g* (*x*))' has a unique analysis in Dummett's sense.

So the question is not even that thoughts may have alternative analyses while sentences (or the senses of sentences) may not: to analyse a thought in the sense of *Bs* §9 just is to analyse a sentence. And again, Frege's procedure presupposes that a unique set of constituents circumscribes all possible decompositions of the sentence.

The same holds, of course, of any putative counter-examples to the uniqueness of analysis.

Perhaps surprisingly, Frege came to hold that the pair in each of the following columns expressed the same thought.[99]

A v B	A & B	A v A	A & A	~~A
B v A	B & A	A	A	A

There is a significant asymmetry between the first two pairs and the other three. The members of each of the former are mere notational variations of one another. As both Geach and Dummett suggest, perhaps a notation could be devised in which the symmetrical connectives of the propositional calculus had no alternative representations so that there would be no difference between 'A v B' and 'B v A', and none also between 'A & B' and 'B & A'.[100] Dummett suggests in this connection a distinction between essential structure and form of representation, such that different forms of representation could express the same essential structure. Different notations could thus express the same essential structure. It can thus be said that Frege's, Peano's and polish notation provide alternative forms of representation of the essential structures of the same sentences. 'A v B' and 'B v A', and 'A & B' and 'B & A', differ in this respect.

The other pairs are trickier. They definitely have different structures. Besides, it follows by transitivity (if equipollence be transitive) that 'A', 'A v A', 'A & A' and '~~A' all express the same thought. Many are inclined to think of this as a "slip of the pen" on Frege's part perhaps due to the false analogy between the first two columns above, on the one side, and the latter three on the other.

But however it may be, it would not be the uniqueness of analysis that were in question, but again whether there were a bijection between sentences of the concept-script and thoughts.

Bermúdez has argued that Frege's views on truth entail that differently structured sentences can express one and the same thought.[101] Frege did indeed say that the sentences 'I smell the scent of violets' and 'It is true that I smell the scent of violets' express the same sense, while 'It is true that ξ' is not senseless.[102]

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[99] Frege 1923.
[100] See Dummett 1981: 328-32 and Geach 1976a: 442-3.
[101] Bermúdez 2001.
[102] Frege 1918: 61.

It is not clear whether Frege would derive Bermúdez conclusion. After all, both sentences have the same transcription into the concept-script. In *Bs*, Frege had already held that the truth-predicate contributed not to content, but to assertion. He would have represented the proposition that *p* as '—*p*' and the proposition that *p* is true as '⊢ *p*'.[103] But he was clear from "On Sense and Reference" on that 'ξ is true' is idle even as regards assertion.[104] Indeed, in the sentence 'if *p* is true, then *q*', '*p*' occurs as much unasserted as it does in 'if *p*, then *q*'.[105] It is rather more likely that Frege would have retracted the equipollence between 'I smell the scent of violets' and 'It is true that I smell the scent of violets' had he ever come to suppose that the sentences had alternative transcriptions into some concept-script. In any case, Bermúdez is right to conclude not against the uniqueness of analysis, but only against the unique pairing of thoughts and sentences of the concept-script, and by implication that thought is not in general linguistic sense.

1.3.3 Converse relational sentences

Amongst the pairs of sentences which bear only differences of tone to one other, Frege counts the following:

> The Greeks defeated the Persians.
> The Persians were defeated by the Greeks.[106]
>
> Hydrogen is lighter than carbon dioxide.
> Carbon dioxide is heavier than hydrogen.[107]
>
> M gave document A to N.
> N received document A from N.[108]

These are what may be called converse relational sentences. Frege thus seems to be committed to the view that, for any sentence that may be formalized as '*aRb*', there is an equipollent one that may be formalized as '$bR^{-1}a$', where $\xi R^{-1}\zeta$ is the converse of $\xi R\zeta$, and $\xi R\zeta$ is not symmetrical.

Converse relations are the same if and only if they are symmetrical. Hence $\xi R^{-1}\zeta$ and $\xi R\zeta$ are different relations: they are different sets of ordered pairs. In Fregean terms, they have different value-ranges: they map different ordered pairs onto truth-values. The relation-words '$\xi R^{-1}\zeta$'and '$\xi R\zeta$' thus have different referents, from which it follows that they have different senses.[109]

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[103] Frege 1879 §3.
[104] Frege 1892a: 34.
[105] Geach 1961: 133.
[106] Frege 1879 §3.
[107] Frege 1879 §9.
[108] Frege 1897: 141.
[109] It has been pointed out to me that the non-synonymy between 'ξ defeated ζ' and 'ξ were defeated by ζ' may not be as straightforward as that between 'ξ is lighter than ζ' and 'ξ is heavier than ζ', or that between 'ξ gave document A to ζ' and 'ξ received document A from ζ'. The suspicion is supposed to apply in general to the senses of those converse relations that are involved in the active and passive transformations of a sentence. This "intuition", however, is grounded in a merely psychological inclination, suggested perhaps by the simplicity of the grammatical rule that allows for the transition between the alternative voices. But the Fregean notion of sense is coordinated with that of reference: to grasp the sense of an expression is to know a way to determine its referent. The sense of a predicate is its

Now, if '*aRb*' and '$bR^{-1}a$' express the same thought, then there is no bijection from sentences of the concept-script to thoughts. The thought expressed by a sentence cannot be its (linguistic) sense.

1.3.3.1 A misdescription

The case is sometimes expressed in a slightly different way, as if '*aRb*' and '$bR^{-1}a$' corresponded to alternative function-argument decompositions of the same sentence. Beaney in particular suggests such a formulation of the matter when he writes, commenting on Frege's *Bs* §9, that

> A relational proposition such as 'Hydrogen is lighter than carbon dioxide' can be analysed not only into an argument and a one-place function (in two different ways) but also into two arguments and a two-place function. Indeed, it might be suggested that this latter analysis is the most basic analysis, since it identifies all the fundamental objects. *But even here there are two possible analyses*, since what is important is not just the objects identified but also the order in which they are represented in a proposition, reflecting the precise relation (i.e., two-place function) involved.[110]

Now, if the passage is not ambiguous and by the word 'analyses' in the italicised clause Beaney means the procedure described by Frege in *Bs* §9, then what Beaney claims is strictly mistaken, for two reasons.

First, as we have seen, "the most basic analysis" of 'Hydrogen is lighter than carbon dioxide' must correspond to an analysis in Dummett's sense, i.e. to a decomposition, not of the sentence, but only of one of its constituent complex predicates. From 'Hydrogen is lighter than carbon dioxide' we can only extract the concept-words 'Hydrogen is lighter than ξ' and 'ξ is lighter than carbon dioxide'; and only from either the relation-word 'ξ is lighter than ζ'.

Now, none of these predicates occurs in 'Carbon dioxide is heavier than hydrogen'. From the latter sentence one can only arrive at 'Carbon dioxide is heavier than ξ', 'ξ is heavier that hydrogen' and 'ξ is heavier than ζ'.

The relation-words 'ξ is lighter than ζ' and 'ξ is heavier than ζ' are obviously distinct. As Geach would say, they are themselves functions, linguistic functions: in fact converse non-symmetrical linguistic relations, which map different sets of ordered pairs of proper names onto different sentences.

The concept-words 'Hydrogen is lighter than ξ' and 'ξ is heavier that hydrogen' are also different linguistic functions: their values differ even for the same arguments. A peculiar thing happens here, though. Although the predicates are different, they have the same referent. The value-range of the function that maps every thing that is such that hydrogen is lighter than it onto the True, and everything else onto the False, is the same as the value-range of the function that maps every thing that is heavier than hydrogen onto the True, and everything else onto the False. The concepts denoted by 'Hydrogen

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criterion of application, that is, the condition that an object must satisfy in order for it to fall under the concept denoted by the predicate. In the case of two-place predicates or relation-words, it is the condition that a set of objects must satisfy in order for them to be related by the relation denoted by the predicate. Now this set must be an ordered pair. It is essential to understand a predicate such as 'ξ defeated ζ' to perceive the difference between 'The Greeks defeated the Persians' and 'The Persians defeated the Greeks'. To know the sense of 'ξ defeated ζ' therefore involves knowing that the order of the objects to which it applies with truth is different from that of 'ξ were defeated by ζ'. This is what gives substance to the general claim that converse relation-words must have different senses if they have different referents.

[110] Beaney 2007: 98. My italics.

is lighter than ξ' and 'ξ is heavier that hydrogen' are thus co-extensive, even necessarily so, and so not two but one.

Now, strictures about Kerry's paradox aside, in order to refer to the concept denoted by both predicates, we can construct a definite description from each with the help of a term-forming operator such as 'the concept of being (such that) ξ'; say:

'the concept of being such that hydrogen is lighter than it';
'the concept of being heavier that hydrogen'.

As a matter of linguistic necessity, the definite description constructed from 'Hydrogen is lighter than ξ' is grammatically more complex than the one formed from 'ξ is heavier that hydrogen', although this complexity has no logical import. As a consequence, in order to avoid the circumlocution, one often chooses to refer to the concept through the definite description 'the concept of being heavier that hydrogen'. And so we have it that, for entirely pragmatic reasons, one may be tempted to use the simpler, more natural phrase, even in contexts where only the more complex one would be legitimate. Frege, for one, does seem to fall into this trap when, concerning the alternative decompositions of 'Hydrogen is lighter than carbon dioxide', he writes that

> In the manner of treatment just indicated, 'hydrogen' was the argument and 'being lighter than carbon dioxide' the function; but we can equally look at the same conceptual content in such a way that 'carbon dioxide' is the argument and 'being heavier than hydrogen' is the function.[111]

Here Frege had of course not yet drawn his distinction between sense and reference. But even if he could be in some sense justified in claiming that the *function* 'being heavier than hydrogen' could be extracted from 'Hydrogen is lighter than carbon dioxide', it would have been nevertheless wrong for him to say that the *predicate* 'ξ is heavier than hydrogen' could. Rather, Frege should have said that the second decomposition of the sentence involved the predicate 'ξ is such that hydrogen is lighter than it' instead.

We can summarize the point in general terms: '$bR^{-1}a$' does not correspond to any possible function-argument decomposition of 'aRb' because:

> (1) we can arrive at the relation-word '$\xi R\zeta$' from 'aRb' and at '$\xi R^{-1}\zeta$' from '$bR^{-1}a$', but in order to pass from '$\xi R\zeta$' to '$\xi R^{-1}\zeta$' we would need, besides predicate extraction, the definition of the converse of a relation;
>
> (2) we can extract from 'aRb' a function-name of the concept that is also denoted by the concept-word '$\xi R^{-1}a$' in turn extracted from '$bR^{-1}a$', namely the predicate '$aR\xi$', but not the concept-word '$\xi R^{-1}a$' itself.

Quite simply put, there is no function-argument *analysis* of any of the sentences 'aRb' and '$bR^{-1}a$' such that one corresponds to a decomposition of the other. That 'aRb' and '$bR^{-1}a$' are equipollent does not imply that there are alternative function-argument decompositions even at the fundamental level of analysis. What it does imply, as we have seen, is that a single thought may correspond to different sentences of the concept-script. The implication is that it is not alternative decompositions that ground the

[111] Frege 1879 §9.

equipollence, but rather conversely, that the equipollence would ground the alternative representations of the thought if it were true. To be fair to Beaney, this is all that he appears to imply when he writes that "If we formalize the proposition as 'aRb' […] then *given the equivalence* […] *above*, we can also formalize it as 'bR^{-1}a' […]".[112] But even here he should have written 'thought' instead of 'proposition', since by the latter he means roughly the same as Geach. Indeed Beaney tells us that he uses

> the word 'proposition' in this paper in (roughly) the sense in which Frege uses the word 'Satz'. It means more than just 'sentence', understood as a mere linguistic expression, but as something like 'sentence with a content'.[113]

In any case, the point is a relevant one to stress, since, as I believe, its neglect may have led to a certain amount of confusion as to what is at stake in these cases.

1.3.3.2 Sullivan on converse relations

Peter Sullivan has tried to meet the challenge on Dummett's behalf, claiming there to be no inconsistency in accepting that (i) '*aRb*' and '*bR^{-1}a*' express the same thought, (ii) '$\xi R \zeta$' and '$\xi R^{-1} \zeta$' have different senses, and yet (iii) analysis is unique.

Sullivan agrees that converse relations are the same only if they are symmetrical. He explicitly opposes Frege's view on the matter to Russell's, for whom, at least for some time, converse relations were in general identical.[114] Sullivan also agrees that for instance 'heavier' and 'lighter' are not synonyms.[115] But then he argues, in what may seem a somewhat desperate fashion, that this

> […] is plainly not a good question. For Frege, a relation is 'incomplete', which means (at least) that to mention a relation is to speak of one thing's standing in the relation to another. So the only sensible question to ask in this case is whether our proposition speaks of something's being heavier than another, or of something's being lighter than another. And the only sensible answer to that question is that the proposition does both, simultaneously. To introduce into Frege's language a symbol allowing us to say that something *x* is heavier than something *y* is automatically to introduce a symbol allowing us to say of *y* that *x* is heavier than it, i.e. that *y* is lighter than *x*. So we do not have to choose whether our single content involves *heavier* or *lighter* as constituent: for it to include the one *is* for it to include the other.[116]

It would be tempting to interpret the quoted passage as claiming there to be no two constituents after all, especially when Sullivan suggests that to introduce a symbol for one of the relations is thereby to introduce a symbol for the other. But this would have the absurd consequence that two expressions with different referents had the same sense in the same context, something that Sullivan could have hardly intended.

What Sullivan appears to be claiming is that converse relation-words express each other's senses. Indeed, he says that "What the above claims to be "the same" are rather what it is for a sentence *to express* a relation and what it is for it to express the

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[112] Beaney 2007: 98. My italics. I have changed Beaney's notation for the converse of a relation from 'R" to 'R^{-1}'.
[113] Beaney 2007: 95.
[114] Sullivan 2010: 99, note 13.
[115] Sullivan 2010: 98.
[116] Sullivan 2010: 99.

converse of that relation."[117] It would thus seem that a sentence that expresses that *aRb* expresses both the sense of '$\xi R\zeta$' and that of '$\xi R^{-1}\zeta$', not because the latter are synonymous, but because "what it is to express the one is what it is to express the other"; and likewise for a sentence that expresses that $bR^{-1}a$.

Sullivan's effort fails for two reasons. The first is that it is *prima facie* implausible, and that it has an air of an *ad hoc* solution. The idea that converse relation-words may not be synonymous while expressing each other's senses has no intuitive appeal beyond the resolution of the tension under scrutiny.

The second reason is instructive in that it shows that the question of whether there are alternative analyses has perhaps not been sufficiently distinguished from the question of whether there is a bijection between thoughts and sentences of the concept-script.

Sullivan's solution provides a possible explanation of why converse relational sentences may express the same thought, while at the same time maintaining that each thought may be paired with a unique set of constituents. The thought that *aRb* is mapped onto the senses of '*a*', '*b*', '$\xi R\zeta$' and '$\xi R^{-1}\zeta$'. In some sense of analysis, therefore, we can perhaps say that the thought has a unique analysis, i.e. corresponds to a unique set of constituents.

However, it still remains the case that a single thought may be expressed by different sentences of the concept-script. We have seen that to say that '*aRb*' and '$bR^{-1}a$' correspond to alternative decompositions of the same sentence is at the very least misleading. What was at stake was rather that if the equipollence held, thoughts could not be paired uniquely with sentences, even if in Sullivan's case that might not imply that thought was not linguistic sense after all. But Sullivan's answer does not touch the latter question at all.

The result is that, on Sullivan's view, each thought expressible as '*aRb*' is overdetermined by its constituents. Presumably, the senses of '*a*', '*b*', '$\xi R\zeta$' would be sufficient to express the thought that *aRb*, but we are told that the sense of '$\xi R^{-1}\zeta$' is also expressed by '*aRb*'.

Apparently Russell once dealt with the question by simply claiming that *because* 'heavier' and 'lighter' are different constituents, 'A is heavier than B' and 'B is lighter than A' must express different, though (necessarily) equivalent, propositions.[118] To argue thus in this connection would be obviously question begging. However, there is no need to argue at all. Frege's claiming that '*aRb*' and '$bR^{-1}a$' are equipollent rests on a rather trivial blunder. To see why, we may begin by considering what Frege said concerning the difference between sense and tone, and the examples he chose to illustrate it.[119]

1.3.3.3 Sense and tone

Frege's distinctions between sense and reference, thought and assertion and sense and tone, had already been drawn, in an intuitive fashion and more or less confusedly, in *Begriffsschrift*. The distinction between sense and reference is by far the most confused, and appears under the guise of "different ways of determining a content" during Frege's

[117] Sullivan 2010: 99, note 13.
[118] Sullivan 2010: 99, note 12. Sullivan does not attribute the same view to Frege, but merely suggests that he would have no difficulties in addressing the same problem.
[119] 'Tone' is Dummett's term. Frege uses alternatively 'illumination', 'atmosphere', 'mood', 'poetic scent', 'shadowing', 'colouring', 'intonation' and 'rhythm'. I do not know if have been exhaustive.

discussion of "equality of content", i.e. identity.[120] On the other hand, it is remarkable how clearly Frege distinguishes conceptual content from judgement,[121] and content from conceptual content,[122] thus anticipating almost exactly his later distinctions between thought and assertion, and tone and sense respectively.

As Dummett suggests, tone is quite an heterogeneous category, and is more easily defined negatively as comprising "disparate components" of meaning "associated only by belonging neither to force nor to sense." Tone does not determine the kind of linguistic act effected, and so is not a force-indicator; it cannot affect the truth-value of what is expressed, and so does not belong to sense.[123]

Perhaps it can be said with some confidence that for Frege the two main purposes of tone are to help communication and, as Dummett says, to define the style of discourse.[124] Frege implies at least as much when he claims that tone covers those aspects of language that serve "to aid the hearer's understanding, for instance emphasizing part of a sentence by stress or word-order"[125], and those that are meant "to act on the feelings and mood of the hearer, or to arouse his imagination".[126] Dummett illuminatingly points to a feature of dictionaries that may be said to indicate the tone of an expression, when, along with a description of its meaning, a word is characterized as e.g. "archaic", "vulgar", etc. It is thus not uncalled for to say generally that differences of tone are rhetorical differences.[127] Accordingly Frege intends that his concept-script, meant to depict only what is essential to the representation of inference, be insensitive to subtleties of tone.[128]

Other than the difference between the active and the passive voices, and converse relational sentences in general,[129] Frege uncontroversially lists as possibly inducing mere differences in tone (i) word-order, and (ii) the replacement of synonyms by synonyms (for instance with different etymologies).[130] Perhaps more disputable examples include (iii) the difference between 'and' and 'but', (iv) that between 'still' and 'already', and (v) the inclusion or omission of adverbs such as 'fortunately' and 'regrettably'.[131]

In any event, Frege is perfectly clear that all these distinctions are alike in being of "such kind as not to be expressed in our symbolism",[132] as Frege says concerning the difference between 'and' and 'but'. The distinction between active and passive is therefore quite anomalous amongst these.

When Frege asserted that 'The Greeks defeated the Persians' and 'The Persians were defeated by the Greeks' had the same conceptual content, he did so in connection with his repudiation of the logical significance of the grammatical distinction between subject and predicate.

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[120] Frege 1879 §8.
[121] Frege 1879 §2.
[122] Frege 1879 §3.
[123] Dummett: 1991b: 121.
[124] Dummett 1991b: 122.
[125] Frege 1918a: 64.
[126] Frege 1918a: 63.
[127] Kneale and Kneale 1962: 479.
[128] Frege 1879 §3. (See below.)
[129] Frege 1918a: 64.
[130] For instance Long and White's translation of Frege 1897: 140 has 'stroll' and 'saunter' for 'walk' and 'cur' for 'dog'; Geach's translation of Frege 1918: 63 has 'steed', 'nag' and 'prad' for 'horse'.
[131] Frege 1918a: 63.
[132] Frege 1879 §7.

> A distinction of *subject* and *predicate* finds *no place* in my way of representing a judgement. In order to justify this, let me observe that there are two ways in which the content of two judgments may differ; it may, or it may not, be the case that all inferences that can be drawn from the first judgment when combined with certain other ones can always also be drawn from the second when combined with the same other judgments. The two propositions 'the Greeks defeated the Persians at Plataea' and 'the Persians were defeated by the Greeks at Plataea' differ in the former way; even if a slight difference of sense is discernible, the agreement in sense is preponderant. Now I call the part of the content that is the same in both the *conceptual content*. Only *this* has significance for our symbolic language; we need therefore make no distinction between propositions that have the same conceptual content.[133]

The context makes it clear that what Frege means in this passage is that his concept-script is insensitive to the distinction between subject and predicate, and in general to "All such aspects of language" that are "merely result of the reciprocal action of speaker and hearer". In all this Frege believes to be following "absolutely the example of the formalized language of mathematics", in which "subject and predicate can be distinguished only by doing violence to the thought."

Now it is all too clear that Frege does also *believe* that the transformation from active into passive belongs to this category of linguistic distinctions, i.e. tone. He makes the point as explicitly as it could be expected:

> The speaker usually intends the subject to be taken as the principal argument; the next in importance often appears as the object. Language has the liberty of arbitrarily presenting one or another part of the proposition as the principal argument by a choice between inflexions and words, e.g. between
>
> active and passive,
> 'heavier' and 'lighter',
> 'give' and 'receive';
>
> but this liberty is restricted by lack of words.[134]

Alas, Frege's belief is mistaken. Frege suggests that there is a convention according to which a speaker may draw the attention of his hearer to what he intends to be the "principal argument" of a sentence (that is, in relation to a possible decomposition of a sentence, what he would take to be the replaceable part). To bring the intended argument into subject-place could be one such device.

Consider the sentences:

(a) The Greeks defeated the Persians.
(b) The Persians are such that the Greeks defeated them.

According to Frege's convention, (a) exhibits 'The Greeks' as the argument of the function 'ξ defeated the Persians'; (b) 'The Persians' as that of the function 'the Greeks defeated ξ'. The pair (a)/(b) thus satisfies Frege's description of the convention.

Sentence (c) below differs from (a) merely as to word-order.

---

[133] Frege 1879 §3. Let me, for the moment, pass in silence over the fact that the criterion of identity of conceptual content expressed by Frege in this passage sanctions the identity between any two sentences that may be alternatively formalized as '*aRb*' and '$bR^{-1}a$'. I note only that the criterion is introduced in order to justify the logical insignificance of the grammatical distinction between subject and predicate, not the synonymy of such sentences.

[134] Frege 1879 §9.

(c) The Persians (*acus*.) the Greeks (*nom*.) defeated.

(c) shows that it is not even necessary to switch subject and predicate in order to focus the attention of the reader on the intended argument: a simple permutation of word-order, common in highly inflected languages, is sufficient. Plausibly, here, too, the intended argument is 'The Persians' for the function 'the Greeks defeated ξ'. The difference between (c) and (b) is that 'The Persians' is the grammatical subject of (b) but not of (c); it is part of the grammatical predicate of both (a) and (c).

Now both predicates 'ξ defeated the Persians' and 'the Greeks defeated ξ' correspond to possible decompositions of (a): they may be obtained from it by predicate extraction. Thus (b) and (c) differ from (a) only in tone, as does, for the sake of completeness, (d) below.

(d) The Greeks vanquished the Persians.

(c) differs from (a) by word-order, (d) by substitution of synonyms, and (b) by permutation of grammatical subject and predicate.

On the other hand, not only does (e) *not* differ from (a) merely in what matters tone: (e) does *not satisfy* the description of Frege's convention.

(e) The Persians were defeated by the Greeks.

That (e) differs from (a) in more than tone is shown by the fact that each has its own transcription into Frege's concept-script, which is just another way of saying that they are not alternative decompositions of each other. If the difference were one of tone, Frege's concept-script would, as he himself said, be insensitive to it.

But besides, (e) does not fulfil what Frege says that it does in relation to (a). (e) presents 'The Persians' as argument, not for the function 'the Greeks defeated ξ' as (b) and (c) do, but for the function 'ξ were defeated by the Greeks'. Now, unlike the former, the latter function does *not* occur in (a). Hence (e) does not present as "principle argument" an argument for a function that corresponds to a possible decomposition of (a).

It is thus not the pair (a)/(e) but the pairs (a)/(b) and (a)/(c), in particular the first, that fit Frege's own description of the role he attributes to the conversion between the active and the passive voices. The grammatical distinction between the active and the passive voices *does* entail a logical one. Such is *not* a device that merely switches subject and predicate without affecting sense, but is indeed one of the devices of natural language to produce the converse of a relation. And as we have seen, since converse relations have alternative transcriptions into the concept script, they do not differ merely in tone.

1.3.3.4 Even good Frege nods at times

I conjecture that Frege (i) mistook (e) for (b), and that he therefore (ii) was unwittingly unaware that (a) and (e) had alternative transcriptions. The upshot is that Frege did not intend the implications of the claim that sentences of the forms 'aRb' and '$bR^{-1}a$' are equipollent.

Mistaking (e) for (b) was no more than an instance of the temptation described in section 1.3.3.1 above, to which we have already seen that Frege did eventually succumb. (e) presents 'The Persians' as an argument for the function 'ξ were defeated

by the Greeks'; (b) presents 'The Persians' as an argument for the function 'The Greeks defeated the ξ'. Now, regardless of the fact that 'ξ were defeated by the Greeks' and 'The Greeks defeated the ξ' denote the same concept, they are distinct predicates. It is just that, for the reasons indicated above, it is more natural to say that the Persians were defeated by the Greeks rather than that the Persians are such that the Greeks defeated them.

It may seem too elementary a blunder for a genius such as Frege not to notice that sentences such as (a) and (e) would have alternative transcriptions into the concept-script. That nevertheless seems to have been the case, especially when we consider Frege's further comparisons with the grammatical distinction between active and passive.

The following excerpt is one of those passages that have often been quoted to support the view that analysis is not unique, or that thoughts are not intrinsically structured. Levine cites it as evidence that Frege did not have "to assume the burden of showing how" sentences such as 'M gave document A to N' and 'N received document A from M' "express the same thought with the same ultimate simple constituents." which is supposed to explain why he did not hold the view that "Each propositional content admits of a unique ultimate analysis into simple constituents."

> a thought can be split up in many ways, so that now one thing, now another, appears as subject or predicate. The thought itself does not yet determine what is to be regarded as the subject. […] Language has means of presenting now one, now another, part of the thought as the subject; one of the most familiar is the distinction of active and passive forms. It is thus not impossible that one way of analysing a given thought should make it appear as a singular judgement; another, as a particular judgement; and a third, as a universal judgement.[135]

Incidentally, Frege does not say that the though itself does not yet determine what is to be regarded as *the argument*; he says 'the subject'. In the light of what has been said before and of what is about to follow, it is clear that what Frege here means is that language has the means to present now one part of a sentence as the intended replaceable part (i.e. as the argument of the intended decomposition), now the other, as subject. Now, as we have seen, the distinction of active and passive is *not* one such means. That Frege believed that the conversion between active and passive did not entail an alternative transcription of a sentence is shown by the text that surrounds Levine's quotation. Thus Levine's conclusion is not without a bit of irony: Frege indeed felt no need to explain why 'M gave document A to N' and 'N received document A from M' differed, though not because he thought that they expressed different constituents yet the same thought, but because he did not see that they expressed different constituents at all.

Frege invokes the distinction of active and passive as a device that merely switches subject and predicate, or better, nominative and accusative. But his examples of sentences that suffer such a transformation are the following:

> There is at least one square root of 4.
> The concept square root of 4 is realized.
> The number 4 has the property that there is something of which it is the square.[136]

[135] Frege 1892b: 199-200.
[136] Frege 1892b: 199.

All three sentences have the *same* transcription into Frege's concept-script – but it is in relation to these that he claims that the difference between active and passive as no logical significance. Now, for reasons that concern Kerry's paradox, this is actually true of all those sentences that are such that their passive voice presents as subject a definite description that (apparently) refers to a function. As Frege tells us, though, whereas "An equation is reversible; an object's falling under a concept is an irreversible relation." [137] There is a plethora of relations that are strictly inexpressible in Frege's concept-script, and that rather belong to, as it were, the formation rules of his system. One such relation is precisely that of falling under a concept. So there are indeed cases of active and passive distinctions to which Frege's concept-script is insensitive. As he says:

> 'to be Φ is a property of Γ'
>
> is just another way of saying:
>
> 'Γ falls under the concept of a Φ'.[138]

Both are transcribed as 'Φ(Γ)'. Note that the active/passive distinction would already fail to have no logical significance if the sentences were instead 'Γ belongs to the extension of the concept Φ' and 'the extension of the concept Φ has Γ as its member'. Since membership is a relation between objects (between objects and extensions of concepts, which are objects), it may be "reversible"; and with the help of Frege's notation for value-ranges perhaps the latter pair of sentences could have alternative representations.

Frege thus appears to have unwittingly generalized to all cases of active and passive, such as (a) and (e) above, a peculiar feature of only a few sentences, namely those that mention relations which are not expressible within his script. But in general, the distinction of active and passive does induce a logical distinction. It is not merely a rule for switching nominative and accusative, as is that used to form (b) from (a). It is the very device that language has of producing the converse of a relational sentence, when there is such a converse. For the reasons given so far, either Frege did not realize such a trivial fact, or, in the cases in which he may have, there was indeed no real relation neither its converse.

On Frege's behalf, it should be said that he might not have been the only philosopher of language to commit the same oversight. On one occasion, Dummett himself cites the transformation of active into passive as an example of tone distinctions.[139] But the instance he chooses is precisely the difference between 'There is at least one square root of 4' and 'The number 4 has the property that there is something of which it is the square'. Dummett does not suggest any kind of restriction. On the other hand, in other places in which he distinguishes tone from sense he does not mention the active and passive. I have not found a pronouncement of Geach's on the matter.

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[137] Frege 1892b: 194.
[138] Frege 1892b: 201.
[139] Dummett 1981: 278-9.

1.3.3.5 Frege's criteria of equipollence

Another reason that may have no doubt reinforced Frege's blunder was of course the fact that pairs of converse relational sentences are sanctioned by any of Frege's criteria of identity of content.

Peter Sullivan has formulated the criterion of identity of conceptual content that Frege expressed in *Bs* §3 in the following terms:

> ϕ and ψ have the *same conceptual content* iff
>
> Γ, ϕ entail χ iff Γ, ψ entail χ.

Here χ is the set of consequences that can be drawn from ϕ and ψ when "combined with certain other judgements", or premises added, Γ. As Sullivan himself notes, and as Dummett and others also had, this is too weak a criterion for Frege's purposes. If the notion of consequence involved is classical, the criterion qualifies as alike in content every pair of necessarily equivalent sentences, such as all arithmetical statements, something that Frege did certainly not intend.[140]

In any case, as I have noted above, Frege's criterion is introduced as means to justify the logical insignificance of the distinction between subject and predicate. The criterion thus entails not only that (a) and (e) have the same conceptual content, but also that (a) and (b) do. This is of course compatible with my claim that Frege confounded (e) and (b).

Frege expressed his better-known criterion of identity of sense only in unpublished writings.

> Now two sentences A and B can stand in such a relation that anyone who recognizes the content of A as true must thereby also recognize the content of B as true and, conversely, that anyone who accepts the content of B must straightway accept that of A. (*Equipollence*). It is here being assumed that there is no difficulty in grasping the content of A and B.[141]

Michael Dummett has expressed it roughly thus:

> ϕ and ψ express the *same sense* iff
>
> necessarily, $\forall$S if S understands both, then
>
> S recognizes immediately that ϕ and ψ have the same truth-value.

The requirement of immediacy is essential to rule out the possibility of inference, in which case the criterion would sanction many pairs of sentences that could be easily derived from each other, again something that Frege would not intend.[142]

This criterion certainly sanctions any of the sentences (a) to (e) as synonymous. However, there are good reasons, and independent ones too, to think that the condition mentioned in the criterion is only necessary, not sufficient, for synonymy. Frege's criterion would thus be best exploited as a criterion of non-synonymy:

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[140] Sullivan 2004: 684.
[141] Frege: 1906c: 197.
[142] Christopher Peacocke has endorsed a similar principle, which he called 'Frege's Principle', claiming it to be "indisputable". Compare Dummett 1991a: 168-76 with Peacocke 1988.

ϕ and ψ express *different senses* if
possibly, $\exists$S, S understands ϕ and ψ, but
S does not recognize immediately that they have the same truth-value.

Besides, as Dummett says commenting on Peacocke's "Frege's Principle", "Frege surely revealed a sure instinct by employing his principle only to negative effect, to demonstrate non-synonymy."[143]

With Frege's weaker criterion of non-synonymy, the only one he ever explicitly applied, it is possible consistently to deny that 'The Greeks defeated the Persians' and 'The Persians were defeated by the Greeks' are equipollent sentences. It is not enough to claim the consistency of such a claim, though: one should positively motivate it. But such a motivation is indeed forthcoming. As a matter of fact, one may wish to describe the relation between these sentences as (in some sense) saying the same thing, while (in some sense) in a different way. The matter could be put, for instance, using Wittgenstein's Tractarian distinction between fact and complex: the circumstance of the Greeks having defeated the Persians is a complex which instantiates (at least) two facts, that the Greeks defeated the Persians, and that the Persians were defeated by the Greeks. It is as if both facts were two alternative descriptions of the same state of affairs.

Bob Hale has defined a notion of a state of affairs intended to capture a weak notion of sense appropriate to characterize Frege's metaphor of content recarving in *Gl* §64.[144] Hale's definition uses in turn Crispin Wright definition of compact entailment. There are plenty of reasons to suppose that Hale's notion of state of affairs is unable to fulfil its intended purpose. However, that should not blind us to the fact that under Hale's definition, all contingent logically equivalent sentences come out as alike in content. That means that Hale's notion appears to capture the coarser notion of content that Frege might have had in mind when he implied that 'A', 'AvA', 'A&A' and '~~A' all expressed the same thought.[145] So too our converse relational sentences come out as equipollent under Hale's notion.

1.4 Amending Geach's account

Dummett has claimed that "To talk about expressions and their structure, we need the notions of part and whole, not those of function and value."

> Given only the value of a function for some argument, it is not possible to recover the function or the argument. For this reason, it is inappropriate to regard either the argument or the function as a *constituent* or *part* of the value, since we naturally suppose that anything is uniquely analysable into its ultimate constituents, and that the parts of a thing may be discerned by scrutiny of it.[146]

Similar remarks would apply to the realm of sense, although that is not our primary focus for the moment. For instance Geach proposed to interpret Frege's talk of parts and wholes in relation to thoughts as an unfortunate means of expression, based on two contentions. The first was that "we really talk only figuratively when we transfer the relation of whole and part to thoughts"; the second, that Frege had already once been

[143] See also Evans 1982: 18-19 and Hale 1997: 100.
[144] Hale 1997.
[145] Frege 1923.
[146] Dummett 1984: 396.

lured by his own metaphor in the case of truth-values to an absurd conclusion.[147] From the exegetical point of view, Dummett takes the fact that Frege never withdrew the thesis that thoughts have parts, as he did in the case of truth-values, as evidence that he must have taken the metaphor more seriously than Geach would have expected him to. It is true that "Dummett's appeal to Frege's authority is here selective"[148], and that Frege did unequivocally state that some parts of thoughts must be themselves incomplete. However, from the logical point of view, there is indeed a reason for supposing there to be an asymmetry between both cases.

The problem has been exposed by Peter Sullivan in the form of a dilemma for the functional model of sentential complexity.[149] The functional model was originally proposed to deal with questions of logical unity. But where the model definitely succeeds, there is in a sense no problem of logical complexity at all. Conversely, where complexity is apparent, the model has no real explanatory power.

What we have here is a sort of incommensurability between two types of complexity: Sullivan calls them respectively *complexity of constitution* and *complexity of designation*.[150] In the numerical case, we do not think of the representation of a number as the value of a function for an argument as an *analysis* of that number. We do not mean to say that a number is in any sense complex or articulated. We have rather provided a means of specifying the number, and thus have only complexity of designation. Likewise, when we say that a truth-value may be the value of a concept for an object, we do not mean to assert that the truth-value is thereby being analysed. That is why the analogy works so smoothly in the case of reference and Frege's explanation of the mode of composition of the semantic values of the components of sentences in functional terms is so satisfactory. This way of understanding the relation between truth-values, concepts and objects is precisely what makes it quite absurd to speak of truth-values as in any way complex themselves.

On the other hand, the kind of complexity which calls for explanation in the case of sentences (and perhaps of senses) is closer to complexity of constitution. Sentences *are* articulated, and since they map thoughts, the latter appear to be so too. The problem is that appealing to the functional model in such cases risks "throwing away the baby with the water." Accepting an explanation of sentential unity in functional terms requires us to turn a blind eye to the fact that what prompted the question of unity in the first place was a perplexity about complexity of constitution, not designation.

Consider Ramsey's problem again. In order to explain how the same sentence could have alternative analyses, Geach argued that there is a way of understanding the combination of the constituents of sentences which allows for the required multiplicity. Indeed, if we regard a sentence as the value of a linguistic function, nothing stands in the way of our conceiving there to be different functions for that very same value. True: but no less true is the fact that we are not inclined to think of the representation of something as the value of a function as an *analysis* of it at all. As in the numerical case, we are no longer inclined to think of linguistic functions and arguments as *constituents* of articulated sentences. We have stepped away from the conceptual framework in which it would be meaningful to speak of an *analysis* of a sentence, as intended.

So even if Geach had good reasons to claim that to speak of parts and wholes in relation to sentences and especially thoughts were at best a metaphor and at worst a

[147] See chapter 3.
[148] Geach 1976a: 444.
[149] Sullivan 1992.
[150] Sullivan 1992: 92.

misleading one, it would not be as yet entirely clear that to speak of functions and arguments in that connection is no less felicitous either.

The point becomes more intuitive as soon as one pursues one strand of the analogy between numerical and linguistic functions.

Just as any number can be represented by an arbitrarily defined numerical function, so may any sentence be conceived as the value of an arbitrarily defined linguistic function. Geach's functional model imposes no restrictions upon the form that the function whose value the sentence is may take. It is thus forced to recognize many representations of a sentence in terms of function and argument as possible analyses of it which we would intuitively not declare as such.

On a natural conception of analysis, one would expect an analysis of a sentence to reveal what is involved in understanding it. Now a simple way to formulate Dummett's objection is to say that the doctrine that to analyse a sentence is to represent it as the value of a function overgenerates. It fails to provide a reasonable account of what it is to understand a sentence because it allows that, for each sentence, there may be many function-argument representations that are unintended analyses of it. Since to grasp a thought is to understand the sentence whose sense it is, the functional model cannot provide an account of what is involved in grasping a thought either. And so Frege's dictum that "We can regard a sentence as a mapping of a thought" would be contradicted.[151]

Geach's model thus appears to entail that we are (i) left with no means to provide an explanation of what it is to grasp a thought, and (ii) bound to be able to grasp a thought in advance of understanding any sentence that expresses it. These difficulties, Dummett says, are each the side of a single coin: the first relates to "the needs of philosophers of language or thought"; the second, to the abilities of language-users and thinkers.[152]

The model provides no interesting or philosophical explanation of what it is to grasp a thought because it is unable to show what is distinctive about those functions which do correspond to legitimate analyses of a sentence, as opposed to those which may be correlated to a sentence as a matter of arbitrary stipulation. The relation between the analysis of a sentence and a thought thus becomes purely extrinsic.

As a result, the supposedly intrinsic connection between the sense of a sentence and the expression of a thought is severed. The senses of the linguistic functions which would correspond to correct analyses are no longer more related to the thought then any other functions whatever, regardless of whether or not they provide an analysis of the sentence. We must therefore be capable of "grasping a thought in advance of any sentence that expresses it," [153] because any sentence could in principle express it.

So on the one hand, an arbitrary combination of linguistic functions and arguments could express the same thought, as long as they yielded the same value. On the other, our access to a thought could not be mediated through language, since sense becomes irrelevant to the expression of thought.

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[151] Frege 1919: 255. One might of course deny that to grasp a thought is to understand a sentence, and have some alternative conception of analysis and of the relation between linguistic sense and the expression of thoughts. But then to analyse a sentence on this model could not be to analyse a thought either, as there would be many "analyses" of a sentence which would not be recognizable analyses of a thought anyway.

[152] Dummett 1981: 268.

[153] For independent, yet of course related reasons, a few authors have indeed provided alternative accounts of Fregean thoughts according to which that they are in fact grasped as unstructured wholes. See for instance Sluga 1980 and Bell 1996.

As Dummett says, the analogy between linguistic and numerical functions does come at a heavy price:[154] it divorces linguistic understanding from the expression of thoughts, by treating the philosophy of language as irrelevant for the philosophy of thought.[155] "Geach, of course, intends no such large consequences."[156]

Incidentally this family of objections is rather unfair to Geach. It rests on an uncharitable reading, or indeed on a misunderstanding, of what a linguistic function is.

> Observe that although $105=2.7^2+7$, the numeral '105', unlike '2.7^2+7', is *not* a value of the linguistic function in question; so in writing down '2.7^2+7' we are mentioning the numerical function $2.\xi^2+\xi$, but in writing down '105' we are not.[157]

Here Geach is quite explicit that there are constraints as to what may be the value of a linguistic function. But there should be no need to be so told.

As Sullivan put it, there are indeed many functions which may yield the value 'Socrates is wise' for the argument 'Socrates'. But that is not quite Geach's doctrine. An accurate way to express it would be rather to say that 'Socrates is wise' is the value for the argument 'Socrates' of that function whose value for any name ***a*** as argument is the sentence [***a*** is wise]. Hence, being determined the base sentence and the range of sentences resulting from it by admissible substitutions, to use Dummett's terms, the set of possible linguistic functions is implicitly circumscribed.

A linguistic function is a pattern discernible along sentences. That is to say that it gives a rule for constructing sentences from expressions. The predicate in 'Socrates is wise' is not 'is wise' but the pattern name-followed-by-'is wise'. Other sentences may be constructed following this pattern; for instance 'Plato is wise'. What this means is that a linguistic function provides a description of a rule to construct a sentence. As Sullivan says, this is not a retraction from his claim that functional complexity "is a complexity of designation rather than constitution":

> For this does not imply that we may not designate something through a description of its intrinsic constitution. It is not universally true that "we must be able to identify the value of a function in advance of knowing that it is the value of that function". For our grasp of the function may be given by principles for constructing its value from any given argument.[158]

The conceptual difficulty pointed out above is merely apparent. True: a sentence is articulated; true, a number is not analysed by its representation as the value of a function; but the constituency of a sentence may be provided in functional terms.

Dummett tells us of another sense in which Geach's conception of analysis could be said to be too broad. Even if the account were able to determine the predicates that are sufficient for understanding, it could not specify those amongst the latter the grasp of whose senses is also necessary for understanding.

Consider the thought that the Earth spins. While grasping the sense of the predicate 'ξ spins' is required to understand the thought, presumably grasping that of the second-level predicate 'The Earth Φ' is not. Rather, understanding the latter

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[154] Dummett 1981: 268.

[155] Not necessarily natural language; the objections apply equally well if only artificial languages are considered.

[156] Dummett 1981: 270. Although other authors might. See for instance Currie 1985. It is not clear, though, that those who deny that thought is sense intend such a strong thesis.

[157] Geach 1961: 144.

[158] Sullivan 1992: 98.

intuitively depends upon a previous grasp of the sense of 'The Earth spins'. But the sentence is the value for either of the linguistic functions: both predicates are, on Geach's account, on an equal footing. The model is blind to their relative priority in understanding.

This is fair enough. But we have seen that Geach's interest lay primarily in decomposition, not analysis. In any case his account of analysis can be easily modified so as to cope with Dummett's distinction.

To analyse a sentence was for Geach to represent it as the value of a linguistic function. This was deficient because it was insensitive to the distinction between simple and complex predicates, and applied best only to decomposition. But Dummett's distinction between analysis and decomposition can indeed be interpreted in Geach's functional terms:

To decompose a sentence is to represent it as the value of a linguistic function extractable from it. To analyse a sentence is to represent it as the value of a composite linguistic function of non-composite linguistic functions. (Complex predicates being composite linguistic functions; and simple predicates non-composite linguistic functions.) We do not need to replace the terms 'function' and 'argument' by 'part' and 'whole', *pace* Dummett.

Indeed, Dummett's distinction between analysis and decomposition, rather than a repudiation of Geach's general conception of analysis, only entails a generalization of the account.

Summary

Since the notion of a linguistic function generalizes that of a predicate, Geach's doctrine of linguistic functions explains how there may be alternative decompositions of a single sentence.

However, I have otherwise concluded with Dummett that analysis is unique. In a word, decomposition presupposes analysis: predicate extraction is circumscribed by the ultimate constituents of sentences.

Most or all of the objections typically raised against Dummett's views often miss the target. They concern not the uniqueness of analysis, but rather whether thought is to be generally construed as linguistic sense, or analogously, whether there is a bijection between thoughts and the sentences of Frege's concept-script.

On the other hand, some of Dummett's critiques of Geach's conception of analysis are, as we have seen, by large unjustified. Geach's account may be amended in the simple manner just described.

Since Frege undoubtedly stands by the possibility of alternatively decomposing a sentence, there is so far every reason to suppose that he would have adhered to Geach's doctrine of linguistic functions "if it had been put to him", as Geach says.

But there is a further role for Geach's doctrine to play. Since the uniqueness of analysis does not entail the completeness of simple predicates, *pace* Dummett, and since we can accommodate the uniqueness of analytical structure within Geach's functional model, as Sullivan suggests, even simple predicates may be thought to be linguistic functions. In the next chapter we will see why they should be so conceived.

2 Are predicates functions?

I introduce the problem of sentential unity, and explain Geach's Fregean solution to it, again in terms of his notion of a linguistic function. If predicates are functions as Geach contends, then they must be unsaturated. I show that Frege's characterization of his function signs invites such an understanding of them.

At the same time, if predicates are unsaturated, then there must be a linguistic version of Kerry's paradox. Indeed, Geach claims that Frege foresaw it. Oliver disagrees, and concludes that predicates are not unsaturated, and hence not linguistic functions. I explain why Oliver's conclusion may be a little too quick.

2.1 The problem of sentential unity

"A succession of nouns only is not a sentence, any more than of verbs without nouns."[159] Why this is so, is the problem of sentential unity.

A sentence is a sequence of words, but not a list of proper names. In order for a sequence of words to be a sentence, i.e. to express a thought, one of the words has to be a verb. It is an obvious point to make that 'Brutus killed Caesar' is a sentence while, say, 'Cato Brutus Caesar' is not because the latter lacks a predicate. The question is in virtue of what is a word a predicate rather than a proper name: in this case, what makes 'killed' not a proper name.

To say that 'Brutus' is a proper name because is denotes an object while 'killed' is a verb because is denotes a concept (or an action, as Plato would say), would still not answer the question, which would reappear under the guise, What makes a word denote a concept rather than an object? Thus the problem of sentential unity has also been termed the problem of the expressiveness of the proposition.[160]

Again, it would be spurious to appeal to grammatical rules in this connection. It could be argued that a sequence of words is a sentence if it is well formed. Whether a word may occur as a verb or as a noun, is dictated by its grammar, or rules of combination. But then a similar question could be asked concerning the relation between grammar and words. What features does grammar endow a word with in order for it to be a verb rather than a name, i.e. to denote an object rather than a concept? The problem could indeed be expressed in Kantian terms, as concerning the conditions of the possibility of syntax.

In Fregean terms, the question is, In virtue of what is an expression a function-name, i.e. a sign for a function, rather than a proper name, i.e. a sign for an object? Peter Geach finds in Frege's writings a definitive answer to the problem of sentential unity based on the notion of a linguistic function. Function-names denote functions and proper names objects, because they are themselves functions and objects respectively.

According to Geach linguistic functions are what symbolize numerical or other functions.[161] A concept-word must therefore be a linguistic function. In 'Raleigh smokes', the two words have "different modes of significance." 'Raleigh' denotes a man by being its name; but 'smokes' denotes a concept, if anything, not by being a "bare word", but by the fact that 'Raleigh smokes' instantiates the pattern name-followed-by-'smokes'.[162] In general, the occurrence of a predicate is not the presence of

[159] *Sophist*.
[160] Potter 2008: 111-5. The problem of sentential unity does indeed bear a close kinship to Bradley's regress, but unlike Potter I reserve the latter term to the problem discussed at the beginning of chapter 3.
[161] Geach 1961: 144.
[162] Geach 1975: 140.

a "quotable part of a sentence", but the fact that the sentence shares a pattern with other sentences. Geach's wording does indeed suggest expressing the point in Wittgenstein's terms. It is not the complex of signs '*Fa*' that is expressive, but it is rather the *fact* about this complex that *F* can be applied to different names, or that '*Fa*' shares the pattern '*F*ξ' with '*Fb*', '*Fc*', etc, that symbolizes that *a* is *F*.

The fact about a sentence that it instantiates a certain pattern corresponds to a rule of formation. The difference between 'John killed Mary' and 'Mary killed John' – namely, who killed who – could not be appreciated if the predicate were just the bare expression 'killed'.[163] In languages such as English, it is a fact about word order that indicates what the predicate is. We know what is the nominative and what the accusative from the fact that the nominative is written before 'killed', which in turn precedes the accusative. But a different grammatical rule could be used for the purpose of indicating that "the first killed the second": instead of word order, declensions for each case could be used. What is essential is that the mode of composition of the sentence be able to express the direction of the relation.

In general, what syntax must be able to do, regardless of the particular rules it chooses for the purpose, is to show the mode of "combination into a whole" of the referents of expressions. The grammar of a proper name allows it to stand for an object only because it makes it be to a predicate what an object is to a function: an argument. The grammar for a concept-word allows it to stand for a concept by making it behave to proper names as concepts to objects, i.e. by being itself a function. Proper names and objects alike behave like arguments, and only thereby can the former stand for the latter; function-names and functions alike behave like functions and only thereby can the former stand for the latter. The role of names is to complete predicates to form sentences, and the role of predicates is to be completed by names to form sentences. When syntax describes the behaviour of a predicate, what it does is to show that it is unsaturated as a function is: indeed it is a linguistic function.

Now, this is possible only because the notions of object and function are themselves modelled upon their linguistic counterparts.[164] It is the fact that proper names behave linguistically like arguments, i.e. complete expressions, that makes them suitable to denote objects, but only because the notion of an object is just that of the non-linguistic correlate of a proper name. Likewise, it is the fact that function signs behave linguistically like functions that makes them suitable to represent functions, but again only because the notion of a function is merely that of the non-linguistic correlate of an unsaturated expression, i.e. of an expression whose role in syntax is that of being completed by other expressions.[165] And so that a name denotes an object follows from its behaviour as a linguistic argument; that a predicate denotes a concept follows from its behaviour as a linguistic function; that a sentence expresses what it does – an object falling under a concept, for instance – follows from these facts about complexes of signs.

It follows, in general, that what a sign *can* it *must* denote. An expression is able to denote an object if its logical behaviour is that of an argument, but if it behaves like a proper name, then it must denote an object. A sign is a symbol for a function if it is a linguistic function – its role that of being completed by names; but if it behaves like a function, then it must denote a function.

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[163] Geach 1975: 148.
[164] See Dummett 1981: 234-48.
[165] One should beware of the interplay here involved between the notions of reference as modelled on the name/bearer relation and as semantic role. See chapter 3.

2.2 Are Frege's function signs unsaturated?

Geach acknowledges that, although his explanation of the notion a function in *Bs* §9 fits only linguistic functions, "Frege never explicitly adopts the view that linguistic functions are what symbolizes numerical (or other) functions; but it seems likely that he would have adopted it if it had been put to him."

As we have seen, it is quite likely that Frege may have fallen victim to the confusion between use and mention in *Bs*. But Frege certainly suggests the doctrine of linguistic functions when he (i) says that function-names are themselves unsaturated; (ii) explains the notion of a function in terms of the notion of an incomplete expression; (iii) claims that an expression becomes unsaturated when it is employed as a predicate.

Concerning an expression such as '$(2 + 3x^2)x$', Frege writes that

> The essence of the function lies in the part of the expression that is there over and above the 'x'. The expression for a *function* is *in need of completion*, *unsaturated*. The letter 'x' serves only to hold places open for a numeral that is to complete the expression, and in this way renders recognizable the particular type of need for completion that constitutes the specific nature of the function designated above. Hereafter the letter 'ξ' will be used for this purpose instead of 'x'.[166]

Although Frege is explicit in calling a function-name "unsaturated", it could be thought that he meant only that an adequate sign for a function should include an indication of its argument-places, in order to remind us that the *function*, not the function-sign, is unsaturated. But a passage from "What is a Function?" connects the unsaturatedness of a function-sign to its capacity to symbolize a function.

> 'sin' requires completion with a numeral, which, however, does not form part of the designation of the function: This holds good in general; the sign for a function is 'unsaturated'; it needs to be completed with a numeral, which we then call the argument-sign. [...] A functional sign cannot occur on one side of an equation by itself, but only when completed by a sign that designates or indicates a number. Now what does such a complex stand for consisting of a functional sign and a numeral, e.g. 'sin 1', '√1', 'log 1'? A number each time. We thus get numerical signs composed of two dissimilar parts, an 'unsaturated' part being completed by the other one.[167]

As Frege himself notes, we write 'sin( )' or 'sin ξ' "only for the exceptional case where we want to symbolize a function in isolation. In 'sin 2', 'sin' by itself already symbolizes the function."[168] Indeed, "To use a function sign in isolation is to contradict the nature of a function, which consists in its unsaturatedness."[169] It would appear, then, that what symbolizes in 'sin' is not its occurrence as a bare expression, but the fact that it occurs in certain relations to other signs. Whether something is a proper name or a predicate depends on their possible substitutions in the representation of inference.

Shortly after, Frege introduces the notion of a function as the non-linguistic correlate of an unsaturated expression.

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[166] Frege 1893: §1.
[167] Frege 1904a: 663-4.
[168] Frege 1904a: 664, note 3.
[169] Frege 1904b:161.

> The peculiarity of functional signs, which we here called 'unsaturatedness', naturally has something answering to it in the functions themselves. They too may be called 'unsaturated', and in this way we mark them out as fundamentally different from numbers.[170]

Frege invariably explains the unsaturatedness of a function in terms of the incompleteness of his function-signs.[171] Frege's notions of object and function are indeed modelled upon the behaviour of proper names and function-signs. And just as unsaturatedness is transferred from symbol to referent, so is it transferred from sense to sign:

> As a mere thing, of course, the group of letters 'and' is no more unsaturated than any other thing. It may be called unsaturated in respect of its employment as a symbol meant to express a sense, for here it can have the intended sense only when situated between two sentences: its purpose as a symbol requires completion by a preceding and a succeeding sentence. It is really in the realm of sense that unsaturatedness is found, and it is transferred from there to the symbol.[172]

A function-sign is expressive only because its employment as a symbol endows it with unsaturatedness, which is only to say that the behaviour of predicates is that of being completed by proper names. And so a predicate is a linguistic function. It is on this very basis that a notion of an objective unsaturated "entity" becomes intelligible: a function just is the incomplete referent of an incomplete expression. It behaves to objects as predicates to names.

A simple corollary of the view that all and only linguistic arguments denote objects and all and only linguistic functions denote functions is of course Kerry's paradox, or the paradox of the concept *horse*.

2.3 Kerry's paradox

The sentence 'The concept *horse* is easily attained' is *prima facie* true and *a fortiori* meaningful. Benno Kerry presented this sentence as a counter-example to Frege's doctrine that the distinction between concept and object is absolute, that is, that nothing can be both an object and a concept, not even in different contexts.

If the sentence is true it is because the concept *horse* is easily attained, that is, the concept *horse* falls under the concept *concept easily attained*. But then the former must be one of the objects that falls under the latter. Thus the concept *horse* is an object. Therefore at least some concepts are at least sometimes objects.

The conclusion follows only if 'ξ is easily attained' stands for a first-level concept, i.e. that 'the concept *horse*' stands for an object. Perhaps surprisingly, Frege agrees. He rather resists the conclusion by denying that the words 'the concept *horse*' ever stand for a concept at all. The concept *horse* may fall under *concept easily attained* precisely because it is an object. The concept *horse* is not a concept, i.e. it is not the case that the concept *horse* is a concept. This is Kerry's paradox.[173]

The three distinctive features of Frege's philosophical logic that contribute to generate the paradox are that

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[170] Frege 1904a: 665.
[171] See also Frege 1891.
[172] Frege 1923: 39.
[173] Frege 1892b: 195.

(1) all and only proper names denote objects;
(2) no concept may be an object; and
(3) the definite article introduces a proper name.

So in the sentence 'The concept *horse* is a concept',

(4) 'The concept *horse*' must be a (non-empty) proper name,

which is entailed by the fact that

(5) 'The concept ξ' is a term-forming operator;[174]

finally,

(6) 'ξ is a concept' must be the name of a first-level concept.

Note, further, that if (6) is the case, the concept denoted by 'ξ is a concept' maps every object onto the False, so that 'The concept *horse* is not a concept' is not only true, but necessarily so.

2.3.1 Frege's solution

It seems extraordinary that Frege so much as accepted Kerry's formulation of his paradox. By the time he wrote "On Concept and Object" Frege had already drawn the distinction between first- and second-level concepts.[175] Clause (6) above is obviously false. If 'ξ is a concept' were a function-name at all it would have to be the name of a second-level concept. Frege's reason for thinking otherwise was that

> the words 'the concept square root of 4' have an essentially different behaviour, as regards possible substitutions, from the words 'square root of 4' in our original sentence; i.e. the [reference] of the two phrases is essentially different.[176]

And so it was only the fact that Frege adhered to (4) that led him to endorse the paradoxical view. Remarkably, Frege never had second thoughts concerning his criterion for a complex expression to be a proper name, i.e. (3). Therefore, in order to reject (4) Frege would have to disqualify 'the concept *horse*' as a well-formed expression in the first place. This he eventually did.

> Indeed we should really outlaw the expression 'the meaning of the concept-word A', because the definite article before 'meaning' points to an object and belies the predicative nature of a concept. It would be better to confine ourselves to saying 'what the concept word A means', for this at any rate is to be used predicatively: 'Jesus is, what the concept word "man" means' in the sense of 'Jesus is a man'.[177]

[174] Frege 1892b: 197.
[175] Frege 1892b: 201-2.
[176] Frege 1892b: 201.
[177] Frege 1892-1895: 122.

Both 'the referent of the concept-word "man"' and 'the concept *man*' point to an object: since they are introduced by the definite article, they cannot perform their intended predicative role. Therefore, they cannot stand for a concept, and *a fortiori* cannot stand for the concept *man*. But an expression such as 'what the concept-word " " stands for' *can* stand for a concept since it *can* occur predicatively, even if not quite idiomatically. Geach summarizes Frege's way out of Kerry's paradox well.

> The result of inserting an English expression in the blank between the quotes in the context: 'what " " stands for', will stand for, *bedeuten*, whatever that very English expression stands for […].[178]

Thus 'what "man" stands for' will stand for the concept *man* since it has that essentially predicative behaviour that 'the concept *man*' lacks: the former is a predicate while the latter a proper name. The sentence 'Jesus is what "man" stands for' (namely, a man) is perhaps not quite idiomatic, but it nevertheless involves no logical error.

Note that the sentence 'What "man" stands for is a concept' must now be ruled out as ill-formed. If it were well-formed, 'What "man" stands for' would have to be a proper name, which is it not, since 'ξ is a concept' would be a first-level predicate, which it cannot be.

Indeed, as Dummett writes, one of the reasons for construing 'ξ is a concept' as the name of a first-level function lies in an illegitimate analogy between expressions formed from the context 'what " " stands for' out of concept-words, and expressions formed from the same context out of proper names.[179] The phrase 'what "Mount Everest" stands for' stands for what 'Mount Everest' stands for, i.e. Mount Everest. The former is a singular term precisely because it is completely interchangeable with the latter. So we can equally well say that Mount Everest is the highest peak in the world, or that what 'Mount Everest' stands for is the highest peak in the world.

By analogy, it might be tempting to think of 'what "ξ is a horse" stands for' as a singular term too. But as soon as we apply the test of interchangeability *salva congruitate*, we can see that the analogy would have led us astray. For 'what "ξ is a horse" stands for' is completely interchangeable with 'ξ is a horse' precisely because it is not a singular term: both may behave predicatively, and so do. But only if 'what "ξ is a horse" stands for' were a singular term could 'ξ is a concept' be a first-level predicate and 'What "horse" stands for is a concept' well-formed. Since 'What "horse" stands for is a concept' is ill-formed, so is its negation, 'What "horse" stands for is not a concept'.

Kerry's paradox can thus be declared nonsense, since (4), (5) and (6) above are false.

There must be, however, a means to say of a particular concept that it is a concept. What is needed is the name of a function whose argument-place should always be filled by first-level predicates, that is, a second-level substitute for 'ξ is a concept'. As Dummett suggests,

> we can construct an expression of natural language which represents a second-level predicate, by containing a gap intended to be filled by a predicative expression, and which, when the gap is thus filled, will always yield a true sentence. A particularly suitable expression would be '… is something which everything either is or is not': the gap is intended to be filled by a predicative expression, and the resulting sentence will

[178] Geach 1976b: 56-7.
[179] Dummett 1973: 213.

then say that the generalized law of excluded middle holds for the corresponding concept.[180]

Indeed the second-level concept

$(\forall x)\ [\Phi(x) \vee \neg\, \Phi(x)]$

maps every concept onto the True, and so 'Φ is what everything either is or is not' is a suitable replacement for 'ξ is a concept'.

Thus instead of saying agrammatically 'The concept *horse* is a concept' or 'What "ξ is a horse" stands for is a concept', we can legitimately say 'What "ξ is a horse" stands for is something which everything either is or is not'; symbolically,

$(\forall x)\ [\text{Horse}(x) \vee \neg\, \text{Horse}(x)]$.

Note that if we now tried to state the negation of 'What "ξ is a horse" stands for is something which everything either is or is not' we would not arrive at a version of the paradoxical 'The concept *horse* is not a concept'. In its stead, we would have rather '$(\exists x)\ [\neg\text{Horse}(x) \wedge \text{Horse}(x)]$', that is 'Something is both a horse and not', which is an outright, non-paradoxical contradiction.

The same holds of course of relations and functions in general.

Now, as Geach says, "Frege thus escapes checkmate at this move: but at the next move there will be a check not so easily escaped."[181] To adapt Geach's example, we could wish and try, in full accordance with Frege's doctrine, to assert, say, that Brutus is not a concept, but an object. But if we say that what 'Brutus' stands for is not what 'ξ killed Caesar' stands for, what we get is at best the negation of 'Brutus killed Caesar'; never, as perhaps intended, that Brutus is not the concept *killed Caesar*. A sentence such as 'What "Brutus" stands for is not identical with what "*killed Caesar*" stands for' is nonsense, as it is equivalent to 'Brutus is not identical with *killed Caesar*'.

Indeed the predicate 'Φ is what everything either is or is not' is not entirely analogous with 'ξ is a concept'. The latter would be true of every concept if it were not a pseudo-predicate; but it would also be false of every object. But since 'Φ is what everything either is or is not' is second-level, it cannot apply to objects at all. Any attempt to say of a particular object that it is not a concept is therefore doomed to fail, since, in general, no name of a second-level function has argument-places of the appropriate type. 'Jesus is not a concept' is the negation of 'Jesus is a concept'; so in order for the former to be (necessarily) true the latter would have to be (necessarily) false. But 'Jesus is a concept' is not contradictory but ill-formed. The proper name 'Jesus' cannot occupy the argument-place of 'Φ is what everything either is or is not'.

We can perhaps render 'Jesus is what everything either is or is not' as

$(\forall x)\ [\text{Jesus}{=}x \vee \neg\, \text{Jesus}{=}x]$

but what this means is not that Jesus is a concept, but that the concept *being identical with Jesus* is a concept. It is the concept-word 'ξ=Jesus', not the proper name 'Jesus', that fits the argument-place.

[180] Dummett 1973: 216.
[181] Geach 1976b: 57.

By the same token, we cannot say of a concept that it is not an object. To assert that *man* is not an object is to deny that *man* is an object. But again 'What "ξ is a man" stands for is an object' is not contradictory but ill-formed. The concept *ξ is an object* maps every object onto the true. It is therefore a first-level concept; a suitable name for it might perhaps be the complex predicate '$\exists x\ x = \xi$'. But such predicate contains no argument-place for concept-words; the argument of a first-level concept cannot be a concept. If we inserted 'man' into the argument place of 'ξ is an object', what we would be doing would be to substitute the concept-word '$\zeta = \xi$' with 'ξ is a man', and get

$\exists x\ \mathrm{Man}(x)$.

But what the latter means is that the concept *man* is not empty, not that the concept *man* is an object. 'Man is an object' should therefore be glossed as 'Some object is a man'.

The phenomenon is of course general. As Geach observes, any attempt to state category-differences in a well-constructed symbolism such as Frege's must fail.[182] Frege acknowledges as much:

> In the proposition 'Something is an object', the word 'something' takes an argument place of the first kind and stands for a proper name. Thus whatever we put in place of 'something', we always get a true proposition; for a function name cannot take the place of 'something'. […] Just as in language we cannot properly speaking say of a function that it is not an object, so we cannot use language to say of an object […] that it is not a function.[183]

So, in a sense, if not Kerry's paradox, then the inexpressibility of some thoughts must be, as Frege said, shifted but not avoided.

2.3.2 "On Concept and Object", footnote 8

Now if Geach's doctrine that predicates are not "quotable bits of language" but linguistic functions is right, then there is a more straightforward sense in which Kerry's paradox can only be shifted. Whenever it is a concept-word that fills the blank in the context

What ' ' stands for,

then it is not quite the expression that is enclosed within the quotation marks that stands for a concept as intended, but rather the function that is indicated by it. We can as little refer to incomplete expressions as we can refer to unsaturated entities, since incomplete expressions are themselves unsaturated. Indeed 'ξ is a horse' and 'ξ is a man' are no less functions – linguistic functions – than their referents. Therefore the same "linguistic awkwardness" is necessarily present in Frege's own solution to Kerry's paradox.

Geach suspects that Frege foresaw the "reduplication on the linguistic level of [his] difficulties about the concept *man*."[184] In a note to "On Concept and Object", Frege wrote, concerning the sentence 'The concept *horse* is not a concept', that

[182] Geach 1976b: 58.
[183] Frege 1902: 136.
[184] Geach 1975: 149.

> A similar thing happens when we say as regards the sentence 'this rose is red': The grammatical predicate 'is red' belongs to the subject 'this rose'. Here the words, 'The grammatical predicate "is red"' are not a grammatical predicate but a subject. By the very act of explicitly calling it a predicate, we deprive it of this property.[185]

For a seeming reconstruction of Kerry's paradox on the linguistic level, suppose that the 'the predicate "ξ is red"' is a non-empty proper name, which it must be by Frege's criterion. Suppose also that 'ξ is a predicate' stands for a first-level concept that maps any proper name onto the False. Therefore, 'the predicate "ξ is red" is a predicate' must be false, which entails that 'the predicate "ξ is red" is not a predicate' must be true.

Now in reality this argument is mere sophistry, as it rests on a confusion between use and mention. Unlike 'ξ is a concept', 'ξ is a predicate' operates on proper names, not on their referents. Therefore, the suitable candidate for filling its argument-place is not the predicate 'ξ is red', but the phrase 'the predicate "ξ is red"'. Hence the conclusion of the argument should rather be that the sentence '"the predicate 'ξ is red'" is not a predicate' is true, which it is.

Thus has Dummett accurately observed that Frege's footnote is ambiguous between claiming that the predicate 'ξ is red' is not a predicate – which is straightforwardly false –, and claiming that 'the predicate "ξ is red"' is not a predicate, which is straightforwardly true.[186] But either way, there is no paradox whatsoever. The formal mode should have presented no difficulties for Frege.

> There is no more paradox in the fact that the expression 'the grammatical predicate "is red"' is not a grammatical predicate than there is in the fact that the phrase 'the city of Berlin' is not a city. In the material mode of speech Frege was forced into such at least superficially contradictory expressions as 'The concept horse is not a concept' […], but when we are talking about expressions, then we have no motive for denying the obvious fact that the predicate 'is a horse' is a predicate, nor for affirming the obvious falsehood that the phrase 'the predicate "is a horse"' is a predicate.[187]

Accordingly, Geach too admitted that Frege "slightly muddied the water" in his footnote.[188] It is irrelevant that the phrase 'the grammatical predicate "is red"' is a grammatical subject. What is relevant is that 'is red' is in this context not identifiable as a logical predicate, i.e. as a linguistic function, because here it can only occur as a logical subject, i.e. as a proper name. In other words, in the context 'the predicate "ξ is red"', the predicate lacks the essentially predicative behaviour that allows it to stand for a concept.

More precisely, the term-forming operator 'the predicate ξ' must be construed as a function that takes predicates as arguments and yields their proper names (by Frege's criterion) as values. Now, if predicates were objects (which they would be if they were bare expressions), then the arguments of 'the predicate ξ' would be objects that could be non-problematically denoted by its values. But suppose that predicates are linguistic functions. Then the values of 'the predicate ξ' cannot refer to them, since they must be proper names, and so cannot name functions. As we have seen, no functions are objects, and all and only proper names denote objects.

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[185] Frege 1892b: 197, note 8.
[186] Dummett 1955: 96-7.
[187] Dummett 1955: 97.
[188] Geach 1976b: 60.

Geach draws an interesting general lesson from this sort of cases: "Once again we see the futility of trying to escape Frege's difficulties by semantic ascent, by talking about words instead of the objects and concepts signified."[189] Geach had written that

> Language, after all, is not something set over against the whole world, like the Divine Mind; languages are part of the world, linguistic facts and structures are facts and structures in the world. This sets a limit to the usefulness of semantic ascent in solving philosophical problems. We cannot solve the problem of universals by talking about the word 'pig' instead of Jones's pigs; for there is exactly the same problem about the relation of the word 'pig' to its tokens as there is about the relation of The Pig to Jones's pigs.[190]

The inability to refer to the concept horse cannot be resolved by talking about the predicate 'ξ is a horse', because the predicate is itself a linguistic function, and it was the attempt to talk about functions that generated the paradox in the first place.

Alex Oliver disagrees with both Geach's and Dummett's readings of Frege's footnote. The footnote occurs in the following text:

> It must indeed by recognized that here we are confronted by an awkwardness of language, which I admit cannot be avoided, if we say that the concept horse is not a concept,* whereas, e.g., the city of Berlin is a city, and the volcano Vesuvius is a volcano. Language is here in a predicament that justifies the departure from custom. The peculiarity of our case is indicated by Kerry himself, by means of the quotation-marks around 'horse'; I use italics to the same end. There was no reason to mark out the words 'Berlin' and 'Vesuvius' in a similar way. In logical discussions one quite often needs to say something about a concept, and to express this in the form usual for such predications – viz. to make what is said about the concept into the content of the grammatical predicate. Consequently, one would expect that what is meant by the grammatical subject would be the concept; but the concept as such cannot play this part, in view of its predicative nature; it must first be converted into an object, or, more precisely, an object must go proxy for it. We designate this object by prefixing the words 'the concept'; e.g.: 'The concept *man* is not empty'.[191]

Oliver suggests that Frege is not comparing the sentences 'The concept *horse* is not a concept' and '"the predicate 'is red'" is not a predicate' on the one hand, with 'The city of Berlin is a city' and 'The volcano Vesuvius is a volcano' on the other. Rather, he is comparing the proper names 'the concept *horse*' and 'the grammatical predicate "is red"' with 'the city of Berlin' and 'the volcano Vesuvius'.[192]

The relevant feature of the comparison is that while the term-forming operators 'the city of …' and 'the volcano …' produce definite descriptions that stand for the exact same referents as their arguments, such is not the case with 'the concept …' and 'the predicate …'. The definite description 'the city of Berlin' stands for what 'Berlin' stands for; and 'the volcano Vesuvius' for what 'Vesuvius' stands for. But the definite description 'the concept horse' stands, not for what 'ξ is a horse' does, which is a concept, but for an object; while 'the grammatical predicate "is red"' stands, not for what 'ξ is red' does, but for the predicate 'ξ is red' itself.

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[189] Geach 1975: 149.
[190] Geach 1975: 142.
[191] Frege 1982b: 196-7. The asterisk indicates where Frege inserted his footnote.
[192] Oliver 2010: 139.

This "shift" from what the arguments stand for to what the corresponding definite descriptions stand for is marked by the use of italics in the word 'horse' and of quotation marks around 'is red'. But as Frege says, there is no need to mark out the words 'Berlin' or 'Vesuvius' in the same way, since no such shift occurs.

Therefore, under Oliver's reading, it is not the case that by using the phrase 'the predicate "ξ is red"' one thereby fails to refer to the predicate 'ξ is red', *pace* Geach. In this respect, 'the predicate "ξ is red"' differs from 'the concept horse'. A definite description may refer to a predicate, though not to a concept, because only the latter is a Fregean function. This is fine for Oliver, since, according to him, Fregean predicates are merely plain expressions with empty places, not linguistic functions, and so Fregean objects.

If such is Frege's intended contrast, then the footnote should be read simply as an attempt to make the case of 'the concept horse' more palatable, by contrasting it with a more familiar case in which something similar already happens. When Frege says that "by the very act of calling it a predicate, we deprive it of this property" he means to assert neither that the predicate 'ξ is red' is not a predicate, nor that 'the predicate "ξ is red"' is not a predicate. He just means that the concept-word 'ξ is red', which ordinarily stands for a concept, now becomes part of the proper name 'the predicate "ξ is red"', which stands for the concept-word itself.

As we have seen, the features that generate the paradox are (i) the absolute distinction between function and object; (ii) the absolute distinction between proper and function-names; (iii) Frege's criterion that the definite article introduces a proper name. Frege achieves his strategy by holding on to (ii) and (iii), claiming that on the linguistic case, something similar already happens as regards these. What is peculiar about the non-linguistic case is that, besides, we cannot really refer to what we intend to.

To summarize, on Geach's reading of the footnote, the "similarity" noted by Frege between the concept *horse* and the predicate 'ξ is red' is that both are functions, and cannot therefore be denoted by proper names. On Dummett's reading, Frege does note a similarity, but an incorrect one, since neither 'the predicate "ξ is red" is not a predicate' nor '"the predicate 'ξ is red'" is not a predicate' are at all paradoxical. According to Oliver, Frege notes only that definite descriptions constructed out of the term-forming operator 'the predicate …' do not stand for the referents of the concept-words that fill its argument place. Such is the case also with 'the concept …', but only the latter leads to a paradoxical result, since only concepts, not their signs, are functions.

2.3.3 Could function signs be objects?

Oliver finds confirmation for his view that concept-words are for Frege Fregean objects in a letter from Frege to Russell.

Russell had expressed a "philosophical difficulty" with Frege's view that "a function name can never take the place of a proper name": according to Russell, that view is self-contradictory,

> For 'ξ can never take the place of a proper name' is a false proposition if ξ is a proper name, but otherwise it is not a proposition at all.[193]

Frege's answer to Russell implies that function names are objects, because they can be denoted by proper names.

[193] Russell in Frege 1980: 134.

According to Frege, Russell's conclusion that 'ξ can never take the place of a proper name' is not a proposition at all if ξ is not a proper name involves a confusion between use and mention. In the quoted sentence, 'ξ' ranges over linguistic items: proper- and function names. If ξ is a proper name, the sentence is indeed always false, since proper names can take the place of proper names. But if ξ is a function name, the sentence is not only meaningful, but true. It would be meaningless only if ξ were replaced with what the concept-word stood for. As Frege writes, "it is correct to say: If 'ξ' is not a proper name, then 'ξ can never take the place of a proper name' is not a proposition." That is, if 'ξ' is not a proper name, then ξ is a function, and 'ξ can never take the place of a proper name' must be meaningless because it involves a first-level predicate.

> Here '"( ). 3 + 4"' – with two sets of quotation marks – takes the place of 'ξ'. While '( ). 3 + 4' is a function name, '"( ). 3 + 4"' is a proper name, and its meaning is the function name '( ). 3 + 4'. […] Here we find ourselves in a situation where the nature of language forces us to make use of imprecise expressions. The proposition 'A is a function' is such an expression: it is always imprecise; for 'A' stands for a proper name. The concept of a function must be a second-level concept, whereas in language it always appears as a first-level concept. […] Instead of using the imprecise expression 'ξ is a function', we can say: '"( ). 3 + 4" is a function name'.[194]

Thus Frege explicitly recognized not only that concept-words could be denoted by proper names, but also that 'ξ is function-name' stands for a first-level concept. It follows from either, singly, that predicates are objects, as Oliver contended.

There is, however, a strong reason to resist such a conclusion.

First, though, it should be stressed once again that to solve Kerry's paradox via semantic ascent is only a partial solution of the paradox anyway. We can choose to say that 'ξ is a man' is a concept-word instead of 'the concept *man* is a concept', just as we can say that 'Jesus' is a proper name. But although we can say that 'Jesus' stands for something, we strictly cannot say that 'ξ is a man' stands for something,

> for a function name cannot take the place of 'something'. […] We cannot properly say of a concept name that it means something; but we can say that it is not meaningless. It is clear that function signs or concept names are indispensable. But if we admit this, we must also admit that there are some that are not meaningless, even though, strictly speaking, the expression 'the meaning of a function name' must not be used.[195]

This amounts to the observation that there cannot be a single relation of denotation. The relation that obtains between proper names and their bearers is a relation between objects. But the relation that obtains between function names and what they stand for must either be a unequal-level function if function names are objects, or a second-level relation between functions if function names are functions. And the same holds for third-level functions and their names. As a consequence, thoughts such as that no proper name stands for what a concept-word stands for are as inexpressible as that no object is a concept.

It is true that the doctrine of linguistic functions would entail a linguistic version of Kerry's paradox if it were true. The textual evidence does appear to count against Geach's supposition that Frege foresaw the reduplication of the paradox on the

[194] Frege 1902: 136.
[195] Frege 1902: 136-7.

linguistic level. Oliver appears to discharge the *modus tollens* and conclude that Frege did not endorse the view that predicates are linguistic functions. But this move is actually reckless. It appears to neglect that it is not only that the doctrine of linguistic functions entails the linguistic version of Kerry's paradox: it is also that doctrine that entails Kerry's paradox in the first place. It is the fact that predicates are functions that explains why all and only proper names denote objects and all and only function names stand for functions. Without the doctrine of linguistic functions, both Frege's criterion that the definite article introduces an expression that must stand for an object, and his insistence that proper names cannot denote functions are left completely unaccounted for.

Summary

Geach's doctrine of linguistic functions provides a solution to the problem of sentential unity. Now, from the standpoint of Fregean exegesis, that proves, of course, nothing. Perhaps Frege had no solution to the problem at all, or had only a mediocre one. As Oliver notes, if he is right that Frege's function-names are merely "plain expressions with empty places", 'incompleteness' simply meaning different things when applied now to referents, now to expressions, then there is no role for the notion of saturation to play in the solution to the problem of sentential unity.[196]

Kerry's paradox is generated by the fact that proper names cannot denote functions. Since this is entailed by the doctrine of linguistic functions, the latter can be conceived as explaining or justifying the paradox.

If predicates are linguistic functions, then they are unsaturated, and so Kerry's paradox is reduplicated on the level of language. It is likely that Oliver is right when, against Geach, he argues that Frege did not foresee this phenomenon. However, the sensible conclusion to draw is not that predicates cannot be linguistic functions for Frege, but that Frege did not recognize the full consequences of such a doctrine, namely, (i) that predicates cannot be named by proper names; (ii) that Kerry's paradox reappears on the linguistic level. Otherwise, we are left with no explanation of why Frege would have been led to consider the paradox in the first place. I am led to believe that Oliver neglected that the doctrine of linguistic functions entails Kerry's paradox.

[196] Oliver 2010: 145-7.

3 Are the senses of predicates functions?

I present the problem of the unity of thought by analogy with the problem that generates Bradley's regress, i.e. the unity of object and concept. The reason is simple enough. Geach's solution to the first reproduces Frege's solution to the second.

I agree with Dummett that Geach's view that the senses of predicates are functions from senses to senses leads to intolerable results. I resist Dummett's conclusion that the senses of incomplete expressions must therefore be objects. I introduce Dejnožka's reasons for believing that, for Frege, senses are neither objects nor functions. Unlike Dejnožka, I conclude that the problem of the unity of thought is either redundant or inexistent.

3.1 Bradley's regress

A famous argument due to F. H. Bradley purports to show that either universals and relations are unreal, or analysis is deceptive.

Consider a simple sentence of the form '*Fa*'. '*Fa*' says that *a* is *F*. We naturally analyse the thought that *Fa* into the object *a* and the property *F*: we say that *a* has *F*. Now, if analysis is not deceptive, *a* and *F* are distinct entities: neither is eliminable nor somehow reducible to the other. But if *a* and *F* are distinct, then some relation other then identity must hold between them in virtue of which they relate to form the complex state of affairs that *Fa*. Let this relation be *R*. *R* could be for instance the relation of instantiation, so that '*a* has *R* to *F*' reads '*a* instantiates *F*' or '*a* falls under *F*'.

Now the same question may of course be asked concerning the complex *aRF*: is *R* something different from *a* and *F*? If it is not, then *R* is not really an extra entity, and the complex *aRF* reduces to *Fa*; if it is, there must be some relation *R′* in virtue of which *R* holds between *a* and *F* so that the new complex may be formed. *R′* may be a third-place relation of instantiation. Again, the question may be repeated, a new relation *R″* introduced, and so on *ad infinitum*. Therefore, since to suppose that properties or universals are real leads to an infinite regress – "Bradley's regress" –, one should conclude that they do not really exist, but must ultimately reduce to objects.

A similar reasoning concerning sentences of the form *aRb* would show that relations are unreal, and in general that in sentences of any form only the objects mentioned are real.

On the other hand, of course, one could think that it is analysis that is deceptive, and that the distinction between properties or relations and objects is, so to speak, a distinction only in thought. In reality, the proposition that *Fa* constitutes an indivisible whole only to be set apart by us beasts without judgement, and whose unity is an entirely internal affair. Thence Bradley's absolute idealist conclusion according to which it is not possible to analyse a thought without somehow transforming or falsifying it. Bradley's regress would haunt Moore and Russell for years, and greatly exercise Wittgenstein.

Frege's doctrine of unsaturation addresses the problem exposed by Bradley's regress. Frege writes:

> Take the proposition 'Two is a prime number'. […] the two parts of the proposition are […] essentially different; and it is important to realize that this difference cuts very deep and must not be blurred. The first constituent, 'two', is a proper name of a certain number; it designates an object, a whole that no longer requires completion. The predicative constituent 'is a prime number', on the other hand, does require completion

> and does not designate an object. I also call the first constituent saturated; the second, unsaturated. *To this difference in the signs there of course corresponds an analogous one in the realm of* [*reference*]*: to the proper name there corresponds the object; to the predicative part, something I call a concept.* This is not supposed to be a definition; for the decomposition into a saturated and an unsaturated part must be considered a logically primitive phenomenon which must simply be accepted and cannot be reduced to something simpler. I am well aware that expressions like 'saturated' and 'unsaturated' are metaphorical and only serve to indicate what is meant – whereby one must always count on the co-operative understanding of the reader. Nevertheless, it may perhaps be made a little clearer why these parts must be different. *An object, e.g. the number 2, cannot logically adhere to another object, e.g. Julius Caesar, without some means of connection. This, in turn, cannot be an object but rather must be unsaturated. A logical connection into a whole can come about only through this, that an unsaturated part is saturated or completed by one or more parts*.[197]

Frege expresses the problem of the unity of a complex such as *Fa* as the impossibility that two "complete" constituents may combine to form a whole. If the constituents of the complex *Fa* hold together at all, then one of them must be "unsaturated". Frege has no need to require there to be a further relation between *F* and *a*. What "binds" them together is the (already) essentially predicative nature of the concept, or property, *F*.

What is curious about Frege's solution is that, like Bradley, he opts for holism; unlike Bradley, Frege retains the reliability of analysis. Functions, of which concepts and relations are a species, are not objects: if they are entities at all, they are not of the same rank as complete entities. Functions are essentially unsaturated: a metaphorical way to say that their nature is that of merely serving as links between objects. The fact that functions by themselves require completion is what explains that to ask for a further explanation of why *a* and *F* do not hold aloof of each other is in a sense to pose a spurious question.[198]

3.2 The unity of thought

Bradley's regress concerns the Fregean realm of reference. But Frege expresses the problem of unity also in connection with the realm of sense:

> […] not all the parts of a thought can be complete; at least one must be "unsaturated", or predicative; otherwise they would not hold together. For example, the sense of the phrase 'the number 2' does not hold together with that of the expression 'the concept *prime number*' without a link. We apply such a link in the sentence 'the number 2 falls under the concept *prime number*'; it is contained in the words 'falls under', which need to be completed in two ways – by a subject and an accusative; and only because their sense is thus 'unsaturated' are they capable of serving as a link. Only when they have been supplemented in this twofold respect do we get a complete sense, a thought.[199]

Even more explicitly, Frege writes:

> The complete part of a sentence I call a proper name, the unsaturated part a concept-name. To the unsaturated part of the sentence there corresponds an unsaturated part of the thought and to the complete part of the sentence a complete part of the thought, and

[197] Frege 1903: 371-372. My italics.
[198] Dummett 1973: 175.
[199] Frege 1892b: 205.

> we can also speak here of saturating the unsaturated part of the thought with a complete part.[200]

> The unity of the whole [compound thought] comes about through the fact that the thought saturates the unsaturated part or, as we can also say, completes the part needing completion.[201]

Although Geach is not concerned "to defend Frege's doctrine that senses are identifiable objects",[202] he suggests what might have been the latter's solution to the problem of the unity of thought. The solution is simple enough, and merely reproduces Frege's solution to Bradley's regress on the realm of reference. The sense of a proper name is able to combine with the sense of a predicate to form a thought only because the latter is unsaturated, i.e. itself a function from senses to senses.[203]

If Geach is right, then, a thought is to be identified with the value of a certain function – a sense-function[204] or a thought-function[205] – for a certain argument. A sense-function is the (incomplete) sense of a predicate, and takes (complete) senses of proper names as arguments and yields (complete) senses of sentences (thoughts) as values. Compound thoughts are the values of sense-functions from thoughts to thoughts. In this case, it is the senses of propositional connectives that are the sense-functions.

Geach is aware that Frege's talk of thoughts as having parts is an obstacle to his proposal, simply because neither arguments nor functions are parts of values of functions. However, for Geach the question is whether Frege should here "be imitated or only charitably expounded (as Aquinas says concerning the Fathers)".[206] Frege did stress that to apply the part-whole relation to thought is really only a metaphor:

> To be sure, we really talk figuratively when we transfer the relation of whole and part to thoughts; yet the analogy is so ready to hand and so generally appropriate that we are hardly even bothered by the hitches which occur from time to time.[207]

Besides, Frege had already been misled by the same metaphor to the absurd conclusion that truth-values were wholes whose parts were concepts and objects, precisely on the ground that the former are values of concepts for objects as arguments:

> One might also say that judgements are distinctions of parts within truth-values.[208]

Fortunately Frege came to retract from this bizarre doctrine:

> […] corresponding to the whole-part relation of a thought and its parts we have, by and large, the same relation for the sentence and its parts. Things are different in the domain of [reference]. We cannot say that Sweden is a part of the capital of Sweden. The same object can be the [referent] of different expressions, and any one of them can have a sense different from any other.[209]

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[200] Frege 1906c: 201.
[201] Frege 1923: 37.
[202] Geach 1975: 150.
[203] Geach 1975: 149-50.
[204] Geach 1976a: 440.
[205] Levine 2002.
[206] Geach 1976a: 444.
[207] Frege 1923: 36.
[208] Frege 1892a: 165.
[209] Frege 1919: 255.

So just as Frege came to recognize explicitly that concepts and objects are not parts of truth-values, perhaps he should have – even if he actually did not – paid closer attention to the "hitches that occurred from time to time", and come to the conclusion that the metaphor of parts and wholes was just as unfortunate in the case of senses as it was in the case of reference.

In any case, nothing apparently stands in the way of our transferring to the realm of sense the model of functional combination that we find in the realm of reference. Geach's overall account thus preserves a near perfect parallelism between language, senses, and references.[210] On the level of reference we have functions (concepts or relations) from objects to truth-values; on the level of language we have linguistic functions from names to sentences; and on the level sense, sense-functions from name-senses – to use Sullivan's term[211] – to thoughts. This picture accords well enough with Frege's saying that "We can regard a sentence as a mapping of a thought".[212]

3.3 Dummett's objections to Geach's account

In order to understand Dummett's critique of Geach's thesis that the senses of incomplete expressions are themselves incomplete, it is necessary to mention, if only superficially, at least some features of Frege's theories of sense and reference.

3.3.1 The theory of reference

Frege's theory of reference is a queer one. A reader caught off guard might simply jettison the idea that predicates refer to concepts and sentences to truth-values as typical philosophical superstition. This reaction is prompted by conceiving reference on the model of the relation that obtains between a proper name and its bearer. But it has long been Dummett's contention, which I here take for granted, that Frege's general notion of reference is best understood in terms of the notion of semantic role.[213]

The semantic value of an expression is that which it contributes to the determination of the truth-value of a sentence. It basically corresponds to what expressions are assigned in an interpretation, in the sense of formal semantics: objects are assigned to names, sets to predicates and truth-values to sentences.[214] The only difference between the modern account and Frege's is that, for Frege, a set would be, not the semantic value of a predicate, but the value-range of its referent. The referents of predicates, as we have seen, have a peculiar nature.[215] A predicate stands for a function, which is unsaturated; but the value-range of a function is an object. To what logicians nowadays call the reference, extension, interpretation or assignment of a predicate, Frege called the value-range of its referent; and he reserved the term 'extension' to the value-ranges of concepts alone.

The bearer of a proper name coincides with its semantic value. For that reason, the conflation between both notions of reference in the case of proper names may pass unnoticed. On the other hand, there is intuitively no sense in which something may be the bearer of a predicate. Properties and concepts in the traditional sense rather correspond to what for Frege is the sense of a predicate. For some time, people doubted

[210] Geach 1976b: 440.
[211] Sullivan 1992: 101.
[212] Frege 1919: 255.
[213] See for instance Dummett 1973: chapters 5, 6 and 7, from which I draw.
[214] Dummett 1973: 89-90.
[215] Dummett 1973: 173.

whether predicates stood for anything, or whether a distinction between sense and reference applied to predicates at all. But such doubts disappear under the notion of reference as semantic role. Still, as Dummett says, it is the analogy with reference as modelled on the name/bearer relation that lends Frege's outlook its "realist flavour", and second-order quantification bears testimony to it.

Similar doubts arise in the case of sentences, although sentences have sometimes been misleadingly taken to stand for, say, facts. Under the notion of reference as semantic role, the view that sentences stand for truth-values is again quite unperplexing. However, Frege did here misconstrue the relation between sentences and their referents as the name/bearer relation. Truth-values were for him logical objects, and sentences only a kind of complex proper names.

3.3.2 The theory of indirect reference

Frege's theory of sense can be only superficially understood if one does not conceive of reference as semantic role.

If one only thinks of reference as modelled upon the name/bearer relation, there is nothing particularly innovative in the fact that expressions may have a sense besides having a reference. That two words with different meanings may refer to the same thing is a common sense observation.

It is only because the truth-values of sentences that report speech cannot be functions of the referents of expressions that the notion of sense becomes relevant to logic. That is to say that in *oratio obliqua*, the semantic value of an expression is not its ordinary reference. Frege calls the semantic value of an expression in *oratio obliqua* its indirect reference. In the case proper names, indirect reference appears to be the manner in which an object is presented to a speaker. To the mode of presentation of an object, Frege calls the sense of a proper name. The distinction between sense and reference is relevant only because it has a role to play in the explanation of the truth-conditions of sentences which involve *oratio obliqua*. Frege's theory has no application to intensional contexts other than indirect discourse, *pace* Kripke.

It is the theory of indirect reference that justifies the significance of sense to logic. But Dummett is certainly right to say that Frege's theory of indirect reference presupposes a prior distinction between sense and reference, and so that Frege's theory of sense cannot be simply reduced to his theory of indirect reference.[216]

3.3.3 The theory of sense

The theory of reference stipulates the association between expressions and their referents or values. It is the theory of sense that provides an account of how such correlation is established.[217]

The sense of a word is thus a means the knowledge of which is sufficient for identifying a particular item as its referent. To know such a means is to grasp the sense of the word. The same referent may of course become associated with different words by different means: and so different words may have different senses and yet the same reference.

The thesis that sense determines reference is in general stronger than merely that expressions with the same sense cannot have different referents; or contrapositively, that expressions with different referents must have different senses; or that sense supervenes

[216] Dummett 1973: 90.
[217] Dummett 1973: 93.

on reference. In general, the sense of an expression is a condition that something must satisfy in order to be recognized as its referent.

Frege certainly held the stronger thesis concerning proper names and sentences. The sense of a proper name is the mode of presentation of its referent.[218] A mode of presentation of an object is a way in which it may be identified as the referent of the proper name whose bearer it is. To grasp the sense of a proper name is thus to grasp a condition that an object must satisfy in order to be identified as the referent of the name.

A thought is the truth-condition of a sentence, i.e. the condition that must obtain in order for the sentence that expresses it to be true.[219] So too can the sense of a sentence be conceived as a condition for determining its referent, i.e. its truth-value.

3.3.4 The truth-condition of sentences

Frege was never as explicit about the senses of predicates. But if the semantic role of an expression is its referent, and if the sense of a sentence is its truth-condition, then the strong reading of the thesis that sense determines reference must hold for predicates too.

To say that the sense of a sentence is its truth-condition is to say that it is the condition that must obtain for the sentence to be true. What satisfies this condition is the referent of a proper name: the thought is true if the object denoted by the proper name falls under the concept denoted by the predicate. But the condition that must be met is precisely given by the sense of the predicate: its criterion of application is the condition that an object must satisfy for the predicate to apply to it with truth. And so the sense of a predicate determines a division between objects, i.e. a mapping from objects to truth-values, that is, a concept. It is the sense of the predicate that gives the truth-condition of the sentence.

Again, the sense of '*Fa*' is the condition that *a* must satisfy in order to fall under *F*. But that condition is the criterion of application of '*F*', i.e. the sense of the predicate: the condition that *a* must satisfy in order to be mapped onto the True. And so, by being the condition of the truth of the sentence, the sense of the predicate determines a mapping from objects to truth-values, i.e. a concept.

Now if, as Geach contends, the sense of a predicate is a function, then we are unable to say the condition of the truth of the sentence is the sense of the predicate. On Geach's view the latter is a function from the senses of names to thoughts. Therefore, if the truth-condition of the sentence were the sense of the predicate, it would have to be a condition satisfied by the sense, not the referent, of a proper name.[220] But we cannot specify what such a condition may be beyond vaguely claiming that it is that condition which yields a thought for the sense of a name as argument.

Dummett's diagnosis is that what is wrong with Geach's account is that it entails that it is the sense, rather than the reference, of a proper name that constitutes its semantic role. It is as if the semantic role of proper names were, in ordinary contexts, their indirect reference. On Geach's conception, the referent of a proper name is left out of the account once it is identified as given by the sense of the proper name, whereas in fact it is the reverse that is the case.[221] It is the sense of the proper name that is left out of the account of the truth-condition of the sentence as soon as its referent is given. The truth-condition is then determined as a condition on the referent, not on the sense, of the proper name.

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[218] Frege 1892a.
[219] Frege 1893 §32.
[220] Dummett 1981: 252.
[221] Dummett 1981: 252-3.

To be fair to Geach, it is not clear that he has in mind the criterion of application of a predicate when he speaks of its sense as unsaturated.[222] However, Peter Sullivan has tried to answer Dummett's challenge on Geach's behalf under that assumption.

According to Sullivan, there is a simple response to Dummett's critique. Dummett is unable to see that to carry the sense of a name to a thought is thereby to carry its referent onto a truth-value, since the sense of a name determines its referent. Sullivan writes:

> It is certainly true that the sense of a predicate, conceived as a function, must "carry us from" the sense of a name to a thought: it must, that is, given some condition that an object must meet to be the referent of the name, yield the condition for a sentence containing the name to be true. But now what meets this description, what will carry us from the first to the second, is precisely a condition which the selected object may or may not meet.[223]

Sullivan's intention is clear. He asserts the identity of Geach's sense-function with the condition of truth of the sentence. The problem is that it is hard to see exactly how that can be done. Note that Dummett had not denied the existence of a function as the one defined by Geach. What he objected was precisely to the identification of such a function with the sense of a predicate.[224] The sense of a proper name does determine its referent; *and* the latter satisfies the truth-condition of the sentence if the sentence is true; *and* that truth-condition is the sense of the predicate; *and* once this is in order, a mapping from the sense of the name to the thought is determined. But it is not at all clear how the truth-condition can be such a mapping, and Sullivan does not go much further in clarifying the issue.

In any case, what must not go unnoticed is that neither Geach nor Dummett advance substantive or independent arguments for his views. In particular, Dummett concludes that the senses of predicates must be objects only because they cannot be functions. In his turn, Geach does not advertise for the virtues of a doctrine according to which the senses of proper names are objects. He concludes that the senses of functions are functions only because some part of a thought must be unsaturated.

But of course that it would not follow that the senses of predicates were objects (or functions) if they were not functions (or objects), if they were neither.

3.4 Dejnožka on sense and object

Jan Dejnožka has argued on a scholarly basis that it is not true in general that, for Frege, anything is either a function or an object, precisely because senses are neither objects nor functions.

Dejnožka states his point primarily against Dummett. That senses are objects, Dummett quite simply claims, follows from the fact that they may be referred to by proper names and definite descriptions. Dejnožka's strategy has a negative part and a positive one. First, he shows that whenever Frege states his criteria for an expression to denote an object, i.e. being a proper name or a definite description, the context restricts the domain to ordinary (or customary, or direct) reference. Second, Dejnožka shows that Frege opposes explicitly objects to senses.

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[222] See Geach 1976a: 445 and Geach 1975: 149-50.
[223] Sullivan 1992: 104.
[224] Dummett 1981: 251.

3.4.1 Customary and indirect reference

It is apparently to a thought that we refer by the name 'Fermat's Last Theorem', and in general to the sense of any expression 'A' by a definite description formed by 'the sense of ξ', such as 'the sense of "A"'. Frege writes that

> the three words 'the concept "horse"' do designate an object, but on that very account they do not designate a concept, as I am using the word. *This is in full accord with the criterion I gave - that the singular definite article always indicates an object*, whereas the indefinite article accompanies a concept-word.[225]

It would appear, therefore, that, given Frege's criterion, 'the sense of "A"' must indicate an object too. But Dejnožka notes that the context of the quoted text is perfectly circumscribed to the realm of "ordinary reference", which includes objects and functions, and nothing else. A few lines before, Frege had made a reference to "On Sense and Reference" only to identify his use of the term 'reference' in the sentence 'The meaning of this word [Venus] is thus something that can never occur as a concept, but only as an object.' Thus Frege appears to be opposing only concepts and objects, on that the use of the definite article can never indicate a concept.

As Dejnožka observes, Frege never speaks of sense at all until much later in "On Concept and Object", immediately prior to his reintroduction of the distinction between concept and object.

> A concept is the reference of a predicate; an object is something that can never be the whole reference of a predicate, but can be the reference of a subject.[226]

The implication would be that the distinction between concepts and objects applies only to the realm of (ordinary) reference, as opposed to sense.

Concerning the reference of 'the sense of "A"', Frege writes:

> In order to speak of the sense of an expression 'A' one may simply use the phrase 'the sense of the expression "A"'. In indirect speech one talks about the sense, e.g., of another person's remarks. It is quite clear that in this way of speaking words do not have their customary reference but designate what is usually their sense. In order to have a short expression, we will say: In indirect speech, words are used *indirectly* or have their *indirect* reference. We distinguish accordingly the *customary* from the *indirect* reference of a word; and its *customary* sense from its *indirect* sense. The indirect reference of a word is accordingly its customary sense.[227]

Dejnožka remarks that Frege's first sentence in this passage "is not an anomalous break in the discussion of reported speech, that is, is not an odd shift of topic to customary sense and customary reference."[228] So if the definite article always indicates an object only when customary reference is concerned, there is no implication from that to the fact that 'The sense of an expression "A"' should refer to an object, since the latter concerns only indirect reference.

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[225] Frege 1892b: 195. My italics.
[226] Frege 1892b: 198.
[227] Frege 1892a: 28.
[228] Dejnožka 2007: 82.

> In other words, I think this sentence indicates an exception to Frege's rule that expressions beginning with the singular definite article refer to objects. The exception is precisely for reported speech, i.e., indirect quotation, which by definition refers to senses, as opposed to direct quotation, which refers to names. […] Dummett is right that singular definite descriptions of the form 'the *F*' *customarily* refer to objects, because *customarily* they logically function as object names. But if I am right, Frege classifies 'the sense of expression "A"' as indirect quotation, and therefore it *has* no customary reference, but only an indirect reference, which is its sense. […] By parity of reason, 'the thought that A' refers to a thought as opposed to an object.[229]

Dejnožka's "key text" in the positive part of his argument is the following:

> A truth-value cannot be a part of a thought, any more than, say, the Sun can, for it is not a sense but an object.[230]

According to Dejnožka, this is the only time Frege ever addresses the question he is posing, namely whether senses are objects: and Frege implies that they are not. Besides, the "category exclusion" is indeed general:

> [Frege] does not say, "for it is not a sense but a *truth-value*." He does not say, "for it is not a sense but an *abstract* object." He does not say, "for it is not a sense but a *customary* object," as if senses were objects after all – indirect objects. (Senses are, of course, indirect *references*; this is what indirect reference is all about.) He says, "for it is not a sense but an object." And if no objects are senses, then no senses are objects.[231]

It could of course be replied that this was one of the "slips of Frege's pen": but that would be rather arbitrary since, as Dejnožka notes, Frege writes what he does "in *the* paper on sense and reference."

Dejnožka reasons similarly not only for senses, but also for forces, ideas, and tones. According to him, none of these are able to function really as either objects or functions, and so can be neither. Dejnožka claims that Frege's "dualism" between objects and functions is only "the tip of the iceberg" in his "linguistic turn". If Dejnožka is right, Frege's commitment to the latter lies indeed deeper, in a tie between ontological categories and linguistic functions more austere than even Dummett supposed.[232] But we do not need to pursue such matters further here.

3.4.2 Unsaturated senses

Dejnožka too addresses the problem of the unity of thought. Senses are neither objects nor functions: still, some must be complete, and some incomplete.

We have seen that Dummett concluded that complete and incomplete senses alike should be objects, because everything is either an object or a function, and no senses are functions. According to Dejnožka, this poses Dummett a dilemma.

One is that Dummett faces a problem similar to Kerry's paradox on the level of sense. Dummett is committed to the view that the definite description 'the sense of the expression "A"' stands for something complete. But if 'A' is an incomplete expression,

[229] Dejnožka 2007: 82-3.
[230] Frege 1892a: 35.
[231] Dejnožka 2007: 89.
[232] Dejnožka 2007: 97.

its sense is incomplete. Therefore 'the sense of the expression "A"', if 'A' is an incomplete expression, refers to a complete sense. Hence the paradox.

Dummett would obviously remove the first difficulty by claiming that incomplete senses are not really incomplete, precisely because they are objects, not functions. But the second problem is that then no part of any thought is really incomplete either, so that no solution to the unity of thought is forthcoming.

Dejnožka concedes that Frege recognizes the need for incomplete senses, and claims Dummett to be aware of the fact.[233] But if incomplete senses are not really complete since they are objects, then they are not complete at all. What is worse, the incompleteness of functions (on the realm of reference) is derivative upon that of senses. It would then seem that, on Dummett's account, functions could not be "really" incomplete either.

Admittedly, Dejnožka does not face the reappearance of the problem of the concept *horse* on the level of sense, since he denies that the phrase 'the sense of the expression "A"' must refer to an object, and so to something complete. The definite description may thus refer to an incomplete sense whenever 'A' is an incomplete expression.[234]

But the problem is that, since Dejnožka denies that senses may be functions, he has no substantial account to offer of what exactly is the difference between complete and incomplete senses. He even asserts that it is not modes of presentation that are incomplete: they are rather the complete constituents of incomplete senses.[235] But then incomplete senses just are whatever they must be in order for a thought to be unified. This makes Dejnožka's doctrine devoid of any intuitive appeal.

3.5 Much ado about nothing

I want to argue that the problem of the unity of thought is no other than either Bradley's regress, or that of sentential unity.

Frege writes:

> For not all the parts of a thought can be complete; at least one must be "unsaturated", or predicative; otherwise they would not hold together.[236]

The first sentence makes plain that at least one "part" of a thought must be unsaturated. Now, that thoughts have parts, we already know, is only a metaphor.

> For example, the sense of the phrase 'the number 2' does not hold together with that of the expression 'the concept *prime number*' without a link. We apply such a link in the sentence 'the number 2 falls under the concept *prime number*'; it is contained in the words 'falls under', which need to be completed in two ways – by a subject and an accusative; and only because their sense is thus "unsaturated" are they capable of serving as a link. Only when they have been supplemented in this twofold respect do we get a complete sense, a thought.

We now learn that the sense of proper names cannot be linked without the sense of a predicative part, for instance given by the predicate 'falls under'. But what it means

[233] Dejnožka 2007: 83-4.
[234] Dejnožka 2007: 83, 90.
[235] Dejnožka 2007: 84.
[236] This and the following excerpts form a continuous whole.

for the sense of 'falls under' to be "thus unsaturated" is that the *words* need to be completed in two ways: by a nominative and an accusative. This is just to say that 'falls under' is a linguistic function in the sense of Geach. A thought arises only when the *words* have been completed in this manner. This means that only sentences – i.e. not lists of words – are able to express thoughts. Thus far, then, the problem of the unity of thought is no other than the problem sentential unity. To say that a part of a thought must be unsaturated is glossed as that any sentence that is able to express a thought must involve a linguistic function.

> I say that what such words or phrases stand for is a relation. We now get the same difficulty for the relation that we were trying to avoid for the concept. For the words 'the relation of an object to the concept it falls under' designate not a relation but an object; and the three proper names 'the number 2', 'the concept *prime number*', 'the relation of an object to a concept it falls under', hold aloof from one another just as much as the first two do by themselves; however we put them together, we get no sentence. It is thus easy for us to see that the difficulty arising from the "unsaturatedness" of one part of the thought can indeed be shifted, but not avoided.[237]

Here the difficulty that "can be shifted, but not avoided" is the difficulty generated by (i) the fact that 'the relation of an object to the concept it falls under' designates not a relation but an object, and (ii) three proper names can no more form a sentence than two can. Therefore the words 'the "unsaturatedness" of one part of thought' in Frege's last sentence can only refer either to that which is denoted by the predicative part of a sentence, i.e. a concept or a relation, or to the predicative part of the sentence itself, or to both. In the first case, the "part" of a thought is just a function; on the latter, it is the binding behaviour of predicates, i.e. a linguistic function. In neither case is the "unsaturated part of a thought" an incomplete sense in some further respect.

3.5.1 Referents as "parts" of thoughts

Concepts can be parts of thoughts no more than objects can. Frege famously stressed the point about objects during his discussion with Russell concerning what could be a constituent of a proposition:

> Truth is not a component part of a thought, just as Mont Blanc with its snowfields is not itself a component part of the thought that Mont Blanc is more than 4000 metres high.[238]

Russell's answer was:

> I believe that in spite of all its snowfields Mont Blanc itself is a component part of what is actually asserted in the proposition 'Mont Blanc is more than 4000 metres high'. […] If we do not admit this, then we get the conclusion that we know nothing at all about Mont Blanc.[239]

[237] Frege 1892b: 205.
[238] Frege 1904b: 163.
[239] Russell in Frege 1980: 169.

But if, from Russell's standpoint, Frege's view allowed us to know too little about Mont Blanc, Russell's own would imply that we knew too much. As Frege wrote in a draft of a letter to Jourdain:

> Now that part of the thought which corresponds to the name 'Etna' cannot be Mount Etna itself; it cannot be the meaning of this name. For each individual piece of frozen, solidified lava which is part of Mount Etna would then also be part of the thought that Etna is higher than Vesuvius. But it seems to me absurd that pieces of lava, even pieces of which I had no knowledge, should be parts of my thought.[240]

There are two concerns implicit in Frege's remarks.[241] One is that it does not follow from something's being an object of thought that we thereby know everything that there is to know about it. This is obviously tied to the possibility of an identity's being cognitively valuable. The other is that abstract objects cannot have concrete parts, or indeed any parts. Wittgenstein would have expressed the point thus:

> The part is smaller than the whole: applied to fact and component part (constituent) that would yield an absurdity.[242]

For Frege, a fact is a thought that is true.[243] Facts and thoughts are, in a sense, abstract objects.

> A thought is something imperceptible: anything the senses can perceive is excluded from the realm of things for which the question of truth arises. […] That the Sun has risen is not an object emitting rays that reach my eyes; it is not a visible thing like the Sun itself.[244]

That objects and concepts cannot be parts of thoughts just means that the relation that obtains between objects, concepts and facts is not of the mereological sort. Therefore none of the above counts as an objection against conceiving a thought as a function of objects, concepts, and their modes of presentation.[245]

A thought is a function of the objects and concepts or relations mentioned by the sentence that expresses it because they determine the thought. But as we know from "On Sense and Reference", they determine it only partially. A thought depends *not only* on the objects and concepts, *but also* on their modes of presentation. I emphasize the conjunction because people tend to think of thoughts as unrelated to the realm of reference for no good reason. Different modes of presentation determine different thoughts; but different objects and different concepts do so too.

We could say schematically that a thought that O is C is a thought that object O, presented in way OW, falls under the concept C, presented in way CW. So a thought is a function of O, OW, C and CW. And this is a pretty straightforward, if loose, sense in which Os and Cs are "parts" of thoughts. Now one of these parts is indeed unsaturated:

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[240] Frege 1914b: 79. Michael Potter (2008: 239) humorously remarks that "When he reflected on simplicity, it seems, Frege's thoughts always turned to mountains."
[241] Potter 2000: 120.
[242] Quoted from Potter 2008: 238.
[243] Frege 1918: 74.
[244] Frege 1918: 61.
[245] For a discussion of the relations between Frege's and Wittgenstein's views on facts, complexes and simplicity, and the plausibility of their mutual influence, see Potter 2008, especially chapters 11 and 27.

C. Therefore, if one of the "parts" of the thought must be unsaturated in order for the thought to hold together, then concepts suffice.

It may be useful to interpret Geach's, Dummett's and Sullivan's views on the matter according to this scheme.

For Geach, the "parts" of a thought were the senses of names and predicates. This would be wrong because it would make a thought be only a function of OW and CW, O and C being left out of the account. Perhaps this may partly be what Dummett's critique of Geach's account is about. In any case, we can see that Geach's model would not be fine-grained enough to appreciate the difference between pairs of thoughts such that the first involves two thoughts that differ only as to OW and CW, and the second involves two thoughts that differ also as to O and C. That is, Geach's model is blind to the fact that thoughts also differ when referents differ, and not only when modes of presentation do.

Sullivan's view could perhaps be interpreted as claiming that the sense of the predicate that denotes C and expresses CW is the function whose arguments are O, OW, C and CW, and whose value is the resulting thought. Now, the question is whether this is intrinsically plausible. I would say that it is not; besides, if I am right, such identification would play no explanatory role either.

Dummett's view thus seems to be closer to what I take to be the truth of the matter. His words are:

> The condition for the truth of the thought has to be viewed as the satisfaction by a particular object, given in a certain way, of a certain condition on it. The condition to be satisfied by the object is itself given in a particular manner, corresponding to the sense of the predicate: but it is a condition on the object, that is, on the referent of the proper name.[246]

So while a thought that O is C is a thought that O falls under C, OW and CW are not left out of the account: O is presented in OW, and C in CW. Indeed, this must be what Dummett means when he says that the senses of names and predicates alike contribute to determining the thought, as much as their referents do to determining the truth-value of the sentence that expresses it.

On this new picture of the relation between thought and reference, that a sentence serves as a picture of a thought just means that to the difference between complete and incomplete signs there corresponds an analogous one in the realm of reference.

3.5.2 The senses of incomplete expressions

To be fair, both Geach and Dummett sometimes write as if they did not distinguish the problem of the unity of thought from the problem of sentential unity.

For instance, Geach identifies Dummett's doctrine of simple predicates as complete expressions as the responsible for his inability to explain the unity of thought.[247] But, as we have seen, what the doctrine of simple predicates cannot explain is sentential unity. Accordingly, Dummett has answered (correctly or not) that his doctrine does not entail that simple predicates are a kind of name, but then conceives his answer as addressing whether the senses of simple predicates are incomplete.[248]

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[246] Dummett 1981: 252-3.
[247] See Geach 1976a: 443-4, 445.
[248] Dummett 1981: 318.

On the other hand, Dummett is mainly right when he writes:

> In abandoning [Geach's account of the senses of predicates as functions] we have no need to deny that the senses of incomplete expressions are themselves incomplete, and we should be unfaithful to Frege if we did so. We have, rather, to recognize that they are incomplete not after the mode of referents, but after the mode of senses. To grasp the sense of an expression is no more than to grasp a means of determining its referent: so to grasp the sense of a predicate is to grasp a means of determining a function from objects to truth-values. It is integral to the understanding of the predicate that we recognize it *as* a predicate, that is, as an incomplete expression standing for such a function. It is in this that the incompleteness of its sense consists; there is no role to be played by an assumption that it is itself a function.[249]

I agree; but, first, it is not clear how to reconcile this passage with Dummett's doctrine of simple predicates, since he here treats predicates indifferently as incomplete expressions. (As a matter of fact, there are a number of other passages in which Dummett implies that the level distinctions that Frege finds on the realm of reference are reproduced on the realm of language.) Second, there is as little point in calling the sense of an incomplete expression incomplete, as there is in calling it a function. We can of course incompletely understand an incomplete expression, or understand it completely, but that is obviously an altogether different matter.

To understand a predicate involves two things. One: to grasp its criterion of application, that is, its sense. Two: to understand it as standing for a concept in the sense of chapter 2, that is, as being itself a linguistic function. There is no sense in which the sense of an incomplete expression is itself incomplete: no further role for the metaphor of unsaturation to play.

Summary

Frege undoubtedly recognizes the problem of the unity of object and concept. The gist of his solution lies in the unsaturated, or essentially incomplete character of the function. Frege also undeniably claims that some part of a meaningful sentence must be predicative. This is the problem of sentential unity, which the doctrine of incomplete expressions construed as linguistic functions is able to tackle. Finally, Frege writes that some part of a thought too must be incomplete.

Geach has interpreted the latter as entailing that the senses of predicates must also be functions, sense-functions. I have agreed with Dummett that this renders unintelligible the explanation of the sense of a sentence as its truth-condition. No interpreter of Frege could ever hold on to this result.

However, Dummett has no better reason to conclude that even the senses of incomplete expressions must be objects, since they cannot be functions. I am convinced by Dejnožka's reasons that senses are neither functions nor objects for Frege.

I have shown that what Frege meant by the problem of the unity of thought might be interpreted as the problem of sentential unity, or as the problem of the unity of object and concept. Indeed, there is a sense in which the referents of function names may be loosely said to be parts of thoughts; namely, thoughts are functions of both referents and their modes of presentation.

I have thus implied that there is no necessity to attach a meaning to the phrase 'the incomplete sense of an incomplete expression'. But I have further suggested that to

[249] Dummett 1981: 270.

insist otherwise may simply be the result of confounding 'the sense of an incomplete expression' with 'the incomplete sense of an expression'. If anything, to grasp the sense of an incomplete expression simply involves understanding it as a linguistic function.

This time I am not sure whether the view that I have been advocating should be seen as exegesis or as emendation of Frege's thought. Frege did write about incomplete senses; but on a closer look, he may have meant no more than that predicates and concepts alike are functions; he can certainly be so glossed. However compelling may the idea that a thought must have an incomplete part which is not a referent seem to be, that may be only the result of the alluring power of the picture of the relations between language, sense and reference generated by the symmetry suggested by Geach's model to hold between the three Fregean realms. Frege's own talk of the "third realm" in an almost mythological tone may surely also be counter-productive to construing the relation between sense and reference in a way such as the one that I have put forward.

Conclusion

In chapter 1, I concluded that Geach's doctrine of linguistic functions could be attributed to Frege for two reasons. First, Frege recognized the possibility of alternatively decomposing a sentence. Second, the doctrine of linguistic functions, by generalizing the notion of a predicate, explains how that it possible. The same sentence may be the value of different linguistic functions, which is to say that it may be conceived as having been put together in more than one way.

However, something had to be amiss with Geach's conception of analysis. Sentences could be values of more linguistic functions than could be extracted from them by a single application of Frege's operation. In the simplest case, a relational sentence may be the value of a relation-word with unrelated argument-places, but the latter cannot be obtained from the former.

Dummett's celebrated distinction between analysis and decomposition is useful to understand what is in question. Any component that is also an ultimate constituent of a sentence will in general be obtained not by a single decomposition, but only via a series of decompositions designed to uncover simple predicates. To such a series, we may call analysis.

On the other hand, decomposition presupposes analysis. Any possible decomposition of a sentence is circumscribed by the set of its ultimate constituents. Therefore, although decomposition is plural, analysis is unique.

The critiques that have been levelled against the uniqueness of analysis are not really convincing. Some rest on Dummett's independent, if not independently motivated, and indeed misconceived doctrine of simple predicates. Others are directed against another thesis of Dummett's, that there is a bijection between thoughts and sentences of the concept-script. But even if Dummett were wrong about the latter, it would still be the case that each sentence had a unique analysis.

In particular, I believe I may have contributed to clarify what is at stake in the challenge apparently posed by the equipollence of converse relational sentences to Dummett's account. Again, even in this case it would not be the uniqueness of analysis but whether there is a bijection between thoughts and sentences of Frege's concept-script that would be in question. However, I have argued that, from the point of view of Fregean exegesis the problem is spurious, because Frege simply did not see that converse relational sentences would have alternative transcriptions into his concept-script. If he did, he would have been unlikely to assert their equipollence.

I may have also helped clarify what are the essential insights of Dummett's account of analysis. I have suggested that the association of Dummett's doctrine with his A and B theses may have been counter-productive to Dummett himself.

In this connection, some of Dummett's criticisms of Geach's conception of analysis have also been unfair. Geach's account can be modified in a simple manner so as to easily cope with Dummett's distinction between analysis and decomposition. To decompose a sentence is to represent it as the value of a linguistic function obtained from it by predicate extraction. If the sentence is not a simple monadic predication, then to analyse it is to represent the sentence as the value of a composite linguistic function of non-composite linguistic functions, i.e. as the value of a complex predicate of simple predicates. What is relevant in Dummett's account can thus be accommodated into Geach's functional model.

Since the uniqueness of analysis does not entail the completeness of simple predicates, even simple predicates may be thought to be linguistic functions, since linguistic functions are unsaturated. In connection with the Dummettian distinction

between analysis and decomposition, Geach's doctrine of linguistic functions therefore plays a double role.

On the one hand, it explains how alternative decomposition is possible, in the manner mentioned above. On the other, it explains how elementary (and other) sentences are expressive, i.e. how simple predicates may denote functions, and proper names objects.

It was in chapter 2 that I introduced the problem of sentential unity. If predicates are linguistic functions, they denote functions because they behave to proper names as functions to objects. On the other hand, it is the fact that Fregean functions can only be understood as the non-linguistic correlates of incomplete expressions that makes this explanation plausible. Geach's solution of the problem of sentential unity is thus highly Fregean in spirit.

The way in which Frege characterizes his function-names indeed invites construing them as being as unsaturated as functions. Although I have not focused on Oliver's interpretation of Fregean predicates, it could be argued, as he does, that calling function-names unsaturated is only a manner of speaking. Incomplete expressions are for Oliver just plain expressions with empty places. As I have remarked, this would leave us without an explanation of sentential unity to ascribe to Frege – but as Oliver says, Frege might have had none.

But if I am right, the real ground for attributing Geach's conception of predicates to Frege lies in the fact that the doctrine of linguistic functions entails Kerry's paradox. True: Oliver may be right that Geach's interpretation of footnote 8 of "On Concept and Object" is too fanciful; and Oliver certainly is right that Frege wrote that function-names could be named by proper names, which they could not were they unsaturated. However, I have argued, this is no sufficient ground to reject the attribution of the notion of predicates as linguistic functions to Frege: only that Frege did not follow the consequences of such an idea in detail. The reason is that, otherwise, Frege would have had no good reason to insist on the very feature that generates Kerry's paradox. His insistence that all and only proper names denote objects would then appear to be only whimsical.

In chapter 3, I claimed that Geach's thesis of sense-functions is independent from the doctrine of linguistic functions. Besides, the problem of the unity of thought is either redundant or inexistent.

It would be redundant if it could be interpreted either as the problem of sentential unity, or as the problem of the unity of object and concept, which have I tried to argue that it could.

It would be inexistent if it were merely generated either by a confusion between 'the sense of incomplete expressions' and 'the incomplete sense of expressions' or by the adherence to an excessively schematic conception of the relation between sense and reference.

In any case, Dummett had accurately pointed out that Geach's view that the senses of predicates are themselves functions is untenable. Sullivan had tried to salvage the latter, to my mind unsuccessfully. But to deny that the senses of predicates are functions does not entail that they are objects since, as Dejnožka argued, senses are neither.

Geach's Fregean functional model of sentential complexity should therefore be corrected in two ways. In the first place, it should incorporate Dummett's distinction between analysis and decomposition, which it may quite unproblematically in the way described above. In the second place, it should recognize the spuriousness of extending the metaphor of unsaturation to the level of sense, since there is no further role for the

latter to play beyond that understanding incomplete expressions involves understanding them *as* incomplete expressions, i.e. linguistic functions. Other than that, both Geach's functional model of sentential complexity and his attribution of the notion of a linguistic function, if only implicitly, to Frege, have every reason to recommend themselves.

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